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Optimization Approaches for Electricity Generation Expansion Planning Under Uncertainty

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OPTIMIZATION APPROACHES FOR ELECTRICITY GENERATION EXPANSION PLANNING UNDER UNCERTAINTY

by

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ABSTRACT

In this dissertation, we study the long-term electricity infrastructure investment planning problems in the electrical power system. These long-term capacity expansion planning problems aim at making the most effective and efficient investment decisions on both thermal and wind power generation units. One of our research focuses are uncertainty modeling in these long-term decision-making problems in power systems, because power systems infrastructures require a large amount of investments, and need to stay in operation for a long time and accommodate many different scenarios in the future. The uncertainties we are addressing in this dissertation mainly include demands, electricity prices, investment and maintenance costs of power generation units. To address these future uncertainties in the decision-making process, this dissertation adopts two different optimization approaches: decision-dependent stochastic programming and adaptive robust optimization. In the decision-dependent stochastic programming approach, we consider the electricity prices and generation units investment and maintenance costs being endogenous uncertainties, and then design probability distribution functions of decision variables and input parameters based on well-established econometric theories, such as the discrete-choice theory and the economy-of-scale mechanism. In the adaptive robust optimization approach, we focus on finding the multistage adaptive robust solutions using affine policies while considering uncertain intervals of future demands.

This dissertation mainly includes three research projects. The study of each project consists of two main parts, the formulation of its mathematical model and the development of solution algorithms for the model. This first problem concerns a large-scale investment problem on both thermal and wind power generation from an integrated angle without modeling all operational details. In this problem, we take a multistage decision-dependent stochastic programming approach while assuming uncertain electricity prices. We use a quasi-exact solution approach to solve this mul-

tistage stochastic nonlinear program. Numerical results show both computational efficient of the solutions approach and benefits of using our decision-dependent model over traditional stochastic programming models. The second problem concerns the long-term investment planning with detailed models of real-time operations. We also take a multistage decision-dependent stochastic programming approach to address endogenous uncertainties such as generation units investment and maintenance costs. However, the detailed modeling of operations makes the problem a bilevel optimization problem. We then transform it to a Mathematic Program with Equilibrium Constraints (MPEC) problem. We design an efficient algorithm based on Dantzig-Wolfe decomposition to solve this multistage stochastic MPEC problem. The last problem concerns a multistage adaptive investment planning problem while considering uncertain future demand at various locations. To solve this multi-level optimization problem, we take advantage of affine policies to transform it to a single-level optimization problem. Our numerical examples show the benefits of using this multistage adaptive robust planning model over both traditional stochastic programming and single-level robust optimization approaches. Based on numerical studies in the three projects, we conclude that our approaches provide effective and efficient modeling and computational tools for advanced power systems expansion planning.

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CHAPTER 1: INTRODUCTION

Optimization is a very important approach in mathematics and operations research. It has been widely applied in a broad area of our lives. Mathematical optimization deals with problems of maximizing or minimizing a function of many variables subject to constraints shown as follows,

$$\min f(\mathbf{x}) \tag{1.1a}$$

$$s.t.\ g(\mathbf{x}) \leq 0 \tag{1.1b}$$

where $\mathbf{x}$ is decision variable vector in $\mathbb{R}^n$. $f(\mathbf{x})$ and $g(\mathbf{x})$ correspond to objective function and constraints of this mathematical programming problem. Different properties of $f(\mathbf{x})$, $g(\mathbf{x})$ and $\mathbf{x}$ defines different types of mathematical programming problems, and accordingly different solution techniques are developed.

Optimization has been successfully applied in a great variety of applications, among which the electricity power system benefit greatly by applying optimizations to resolve a large number complex problems in both planning and operations.

1.1 Optimization Models in Long-term Electricity Generation Expansion Planning

Electrical power system is a extremely complex system. Thousands of electrical components are operated, controlled and managed within the electric system to generate, transmit, and supply the electric power. It forms a multi-level network that connects original energy supplies to the ultimate consumers.

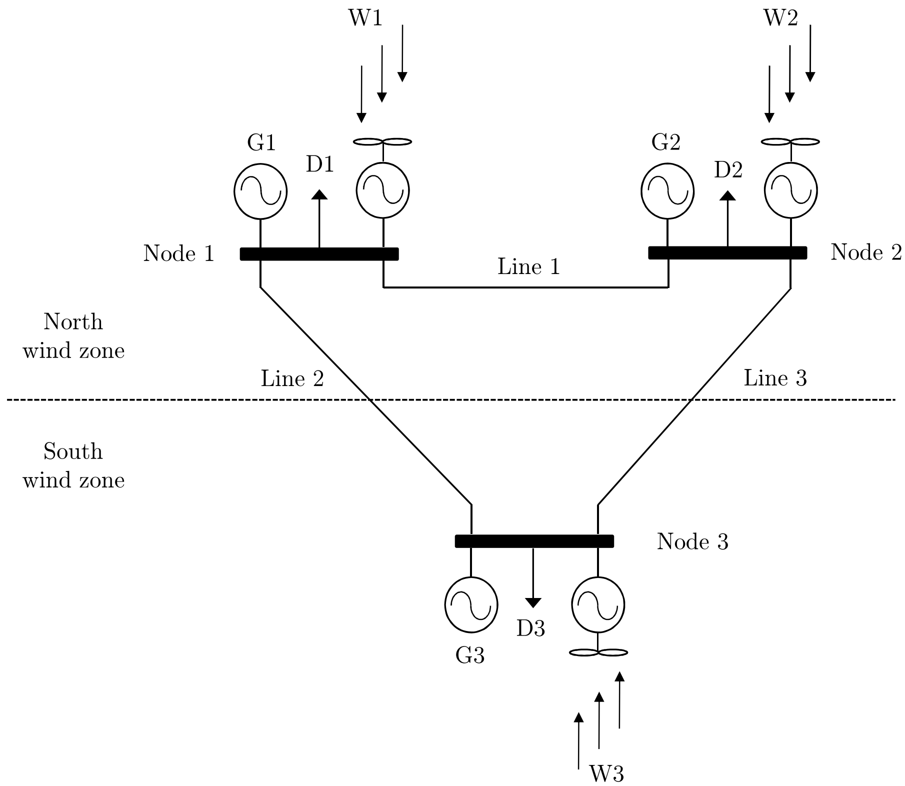

Figure 1.1: IEEE 3 bus power system

Figure 1.1 displays a 3-bus power system which has 3 nodes and 3 arcs. Each node represents a region that supplies or consumes the electricity. Power generators are connected to the network at certain nodes. In this network, there are two forms of power generators: conventional power generator labeled with G and wind power generator label with W. The amount of electricity generated by each unit is restricted by the capacity of each generator, it is also restricted by capacity factor which represents the average ratio of currently installed capacity that can be utilized for generation. Each arc represents a transmission line that transmit electric energy from one region to another. The amount of electricity being transmitted is restricted by the limitation of transmission lines. The demand or load is connected to the nodes with the label of D. The users demand is satisfied by properly operating all the generators and supplying the electricity without violating all the technical restrictions. This 3-bus power system forms a network that addresses three major aspects of electric power system, i.e. generation, transmission and distribution.

In this dissertation, we study a specific type of optimization problems for the power system, which belongs to long-term capacity expansion planning problem. Capacity expansion planning seeks the maximum-profit within the process of expanding the electricity generation facilities to meet the rising demand for the electricity services [2]. Attributing to the long-term planning horizon (15 to 20 years) of the capacity expansion, some of the key parameters are uncertain, including investment cost, electricity price and user demands. Hence, it has been suggested that uncertainty should be considered to achieve effective expansion planning [3]. Another important source of uncertainty attributes to the rapid growth of renewable energy sources in the market. Figure 1.2 shows a clear trend of the growth of renewable energy, especially for the wind power which has a growth rate for more than 15% every year. However, due to the variable and uncertain behavior of wind, the generation planning of a wind farm is still a difficult issue. In order to make most effective and efficient investment decisions, how to deal with increasing uncertainty in the generation planning that involves large-scale electric power is an urgent and challenging task.

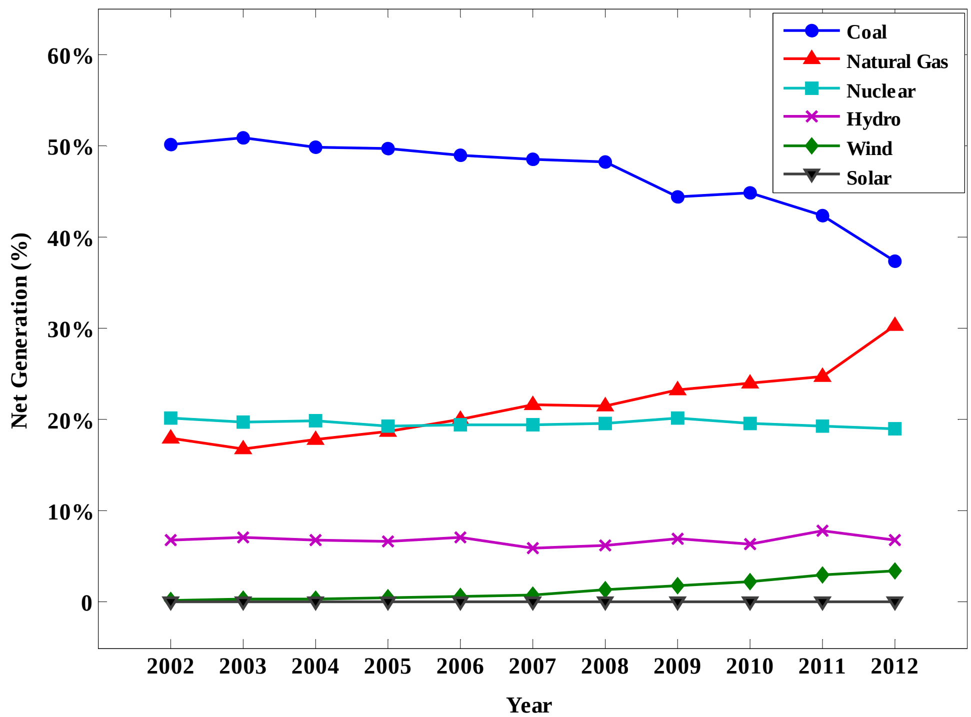

Figure 1.2: Power generation from different energy source

In the following, two popular approaches, i.e. stochastic programming and robust optimization, are introduced to address the uncertainty difficulties within the long-term electricity generation expansion planning.

Stochastic programming is one of the most popular optimization approaches that deal with uncertainty. The uncertain parameters are assumed to be unknown before they are realized at some time point. It aims at finding a solution which maximize or minimize the expected value of all future outcomes, which has following general formulation,

$$\begin{aligned} \min \quad & c^T x + E[Q(x, w)] \\ s.t. \quad & Ax \geq b \end{aligned}$$

where w is an random vector, c and x are the cost vector and decision vector. $E[Q(x, w)]$ is the expected future cost of $Q(x, w)$, which is the cost of decisions made after the uncertainties unfold/

Traditionally, the probability distribution of the stochastic optimization model is predetermined before the uncertainty unfolds, shown as follows,

$$Prob^{\xi} = \kappa(\xi), \tag{1.2}$$

where $\kappa(\xi)$ is a known distribution, such as normal, binomial, Weibull and so on. This is true for the majority stochastic optimization problems. However, there exists the reality that the probability of making decision is affected by the decision itself. For example, in the stock market, the probability of purchasing the stocks is affected by the price of stocks. The price of the stock is also affected by the purchasing decision itself. Hence, the probability of purchasing a stock is affected by the purchase decision. This simple example illustrates the fact that the future uncertainties are not only affecting but also affected by the current decision. In particular, the probability distribution can be dynamically adjustable according to the decisions, i.e. decision-dependent probability. The decision-dependent probability has the formulation as follows,

$$Prob^{\xi}(x) = f(x, \xi), \tag{1.3}$$

where x is decision variable and ξ represents uncertain data.

Nevertheless, the stochastic programming approach has its limitations. For example, in some cases, we need to seek the “safest” solution among all the uncertain data. In the following, we are going to introduce the robust optimization that deals with uncertainty using a different approach.

Unlike the stochastic optimization that uses probability distribution to represent the chance uncertainty, robust optimization considers the uncertainty in a different angle. The robust optimization seeking an “immunized against uncertainty” solution to an uncertain problem and the objective follows the “worst-case-oriented philosophy” [4]. It is formulated as a collection of linear programs of a common structure with the data varying in a given *uncertainty set*. It is generally formulated as follows,

$$\min_{x} \left\{ \sup_{(c,A,b)\in\mathcal{U}} c^T x : \; Ax \leq b, \; \forall (c, A, b) \in \mathcal{U} \right\} \tag{1.4}$$

where (c, A, b) refers to the uncertain data, x is the decision variable vector, $\mathcal{U}$ is a given uncertain set. The original objective is acquired by quantify quality of a robust feasible solution x by its largest value $\sup\{c^T x : \; Ax \leq b, \; \forall (c, A, b) \in \mathcal{U}\}$.

In the robust optimization model, *all* of its decision variables must be determined before the actual realization of the uncertain data [5]. This type of variables that represent the decision made before the realization of uncertain data are called “here and now” decision variables. However, there are some cases in reality that some of variables are able to tune themselves to varying data, that represents “wait and see” decisions. The framework that incorporates this adjustable feature is call *Adaptive Robust Optimization*, which is a extension of Robust Optimization.

For linear structure problems, the linear robust optimization approach is also denoted as robust counterpart (RC) and the linear adaptive robust optimization approach is denoted as adjustable robust counterpart (ARC) [5]. The formulation of ARC of the uncertain linear programming is

$$(ARC) \; : \; \min_{u} \left\{ c^T u : \forall (\xi \equiv [A, b, c] \in \mathcal{Z}) \; \exists v \; : \; Uu + Vv \leq b \right\} \tag{1.5}$$

In contrast, the Robust Counterpart (RC) is formulated as:

$$(RC) \; : \; \min_{u} \left\{ c^T u : \exists v \; \forall (\xi \equiv [A, b, c] \in \mathcal{Z}) \; : \; Uu + Vv \leq b \right\} \tag{1.6}$$

It is obvious to notice that ARC is more flexible than RC with larger robust feasible region. ARC takes advantage of the nature of *adjustable* variables that the decisions can be made until the uncertainty is (partially) unfolded. Thus, ARC enables a better optimal value while still satisfying all possible realizations of the constraints. However, on the other hand, this flexibility brings in computational issues for ARC. Unlike that most RC problems are computationally tractable, most of ARC problems are computationally intractable, i.e. they cannot be solved efficiently [5]. To make the ARC problems solvable, the decision rules of *adjustable* variables need to be specified and restricted. One of the most popular approach is the *affine* policy, which will be discussed in Chapter 4.

If we try to compare Stochastic optimization (SO) against robust optimization (RO), we may notice following differences. In SO, the uncertain numerical data are assumed to be random and follow a certain (usually discrete) probability distribution. Therefore, the number of possible outcomes is finite. Whereas, the uncertain data in RO usually falls within a continuous set, where the number of possible outcomes is infinite. Another important difference between SO and RO is the different goal. SO solves for the optimal of the expected value of uncertain outcomes. The RO, on the hand, solves for the optimal based on the "worst-case-scenario". Its solution must satisfy even the worst uncertain data. Therefore, worst-case-oriented RO approach is more conservative than the SO.

1.2 Outline of this Dissertation

This dissertation is motivated by real world arising operations research problems in the electrical power system. It aims at solving the long-term capacity expansion planning problems with renewable energy under uncertainty. Serval mathematical optimization models are presented and various advance solution algorithms are developed to deal with computational difficulty of each research problem.

The structure of this dissertation is organized as follows. In Chapter 2, we introduces decision dependent stochastic programming model for long-term power generation expansion planning problem. It tries to solve for the maximum profit in capacity expansion when a large amount of wind power is involved. This model uses multistage decision dependent stochastic programming model to address the price uncertainty. The decision-dependent feature enables the probability distribution of stochastic programming to be dynamically adjustable according to the optimization decisions. We employ a quasi-exact solution approach to deal with bilinear constraints and thus transform nonlinear model into mixed-integer linear programming (MILP) model. The wind penetration, investment decisions, and the optimality of the decision dependent model are evaluated in a series of multistage case studies. Chapter 3 investigates the decision dependent multistage stochastic programming for long-term generation expansion planning problem in the market framework. This model seeks for the optimal sizing and siting for both thermal and wind power units to be built to maximizing the expected profit for a profit-oriented power investor. The proposed formulation is based on the bilevel framework that includes an upper-level stochastic expansion planning problem and a collection of lower-level problems that solves for optimal power flow (OPF). The optimal power flow problem solves for the minimum generation cost while considering the technical details of local power network. Transformation and decomposition approaches are developed to overcome the computational challenges of this optimization model. Extensive numerical experiments are conducted to analysis our model and solution algorithm. Chapter 4 studies the generation expansion planning problem with multistage adaptive robust optimization which aims at finding the best solution that satisfies the worst-case-scenario. In this model, the investment decisions turn out to be *adjustable*. In order to manage the computation intractability of the adaptive robust optimization model, a simplified affine policy is applied. Numerical experiments are conducted to study the performance of the proposed model with comparisons to existing approaches. Chapter 5 concludes the dissertations.

CHAPTER 2: DECISION DEPENDENT STOCHASTIC PROGRAMMING APPROACH FOR POWER GENERATION EXPANSION PLANNING [1]

2.1 Introduction

In recent years, renewable energy sources grows rapidly due to the fact that conventional power generation has become a main source of air pollution, and thus face a great challenge of maintaining a sustainable future environment. The development of renewable energy especially wind power generation becomes a potential solution to tackle this challenge. It is considered as an alternative to fossil fuels, due to its incomparable features of being plentiful, renewable, widely distributed and produces no green house gas during operations [6]. However, due to the variable and uncertain behavior of wind, the generation planning of a wind farm is still a difficult issue. It becomes more complicated when wind power is present in a large scale and long term planning. In order to make most effective and efficient investment decisions, how to deal with increasing uncertainty in the generation planning that involves large-scale electric power is an urgent and challenging task.

In this chapter, we propose a long-term planning model through a multistage, decision-dependent, stochastic nonlinear programming approach. We take advantage of the decision-dependent process where the probability distributions of electricity prices depend on the key decision variables: the installed capacities of different types of generation assets.

[1] Y. Zhan, Q. Zheng, J. Wang, P. Pinson. A Decision Dependent Stochastic Programming Model for Power Generation Expansion Planning with Large Amounts of Wind Power, accepted at *IEEE Transaction of Power System*.

2.2 Literature Review

To address these uncertainties, stochastic programming is one of the most popular approaches applied in power system generation planning problems [7], where uncertainties are described by random variables with some predetermined probability distributions. A lot of research has explored stochastic programming approaches not only in the short term generation operation phase [8], but also in the long-term planning phase [9]. Among the many research endeavors pursued for long-term expansion planning under uncertainty, Ahmed et al. [10] addressed a multi-period investment model for capacity expansion in an uncertain environment including uncertain demand and cost parameters, as well as economies of scale in expansion cost. Kennedy [11] has estimated the benefits of large penetrations of wind power with the consideration of stochastic interaction among wind power variability, electricity demand, and the operation of other generators on the power system.

However, most previous works were designed to deal with *exogenous uncertainty* [12], where the probability distributions of uncertain factors are pre-determined and fixed before the optimization process. In other words, the stochastic programming models with *exogenous* uncertainties, such as electricity prices, are usually formulated based on the assumption that future random electricity prices are independent of the investment decisions at the current stage. However, in real-world generation planning, the decision variables in the current stage also play an important role influencing the future uncertainties. The study in [13] shows that the decisions on power plant expansion are affected by several variable criteria including capital costs, current costs, budget deduction, and electricity prices. It is discovered in [14] that different risk-aversion levels result in different investment strategies in wind facilities. In the study in [15], the maximum social welfare is achieved when the electricity price is varying according to a user's energy demands. It is shown in [16] that different installed capacities of wind power will influence the entire power system, especially when a large amount of wind power is involved. The probability distribution of future electricity prices is affected by the level of wind power. All of these research findings indicate that decision variables play an important role in determining the uncertain process at later stages. Hence, in order to consider the *endogenous uncertainties*, we adopt a decision-dependent approach that takes into account decision variables in determining the distributions of the uncertain process. In the operations research field, several studies have utilized this decision-dependent approach to deal with *endogenous* uncertainties. Among the many approaches, a hybrid mixed-integer disjunctive programming approach has been presented in [12] to address a class of stochastic programs with decision-dependent uncertainties. In [17], a decision-dependent approach was applied to a mixed-integer stochastic programming model where the timing of information discovery can be influenced by decisions.

Our proposed model contains bilinear terms that make the optimization process computationally very challenging. This model, known as the bilinear program (BLP), belongs to the class of hard nonconvex nonlinear programs where functions are twice continuously differentiable [18]. Previous studies proposed many theoretical and algorithmic approaches for acquiring the optimality of BLPs, such as deterministic branch-and-bound [19], branch-and-contract global optimization algorithm [20], reformulation-linearization technique [21], Lagrangian relaxation [22], automatic symbolic reformulation procedure [23], linear cutting plane algorithms [24], effective heuristic algorithms [25] and etc. However, there are also limitations among some of the existing approaches that prevent their direct applications to our model. For example, some approaches only work with special BLPs (e.g., disjoint BLP [19], BLP with nonlinear constraints [20, 24]), some approaches only converge under certain conditions (e.g., the zero duality gap conditions for Lagrangian relaxation [22]), some approaches may not always converge to a global optimum [25]. Nevertheless, the quasi-exact solution algorithm in [26] uses a straightforward linearization mechanism that does not have the aforementioned limitations.Borrowing the method that a modern computer represents any fractional number by using binary variables, the quasi-exact approach ensures that the MILP (Mixed Integer Linear Programming) problem is equivalent to the original problem when a large number of binary variables are used to ensure the accurate representation of the fractional numbers. Hence, the new resulting MILP (Mixed Integer Linear Programming) problem can be solved conveniently and efficiently by using any off-the-shelf MILP commercial solver, but still attains a high level of accuracy as shown in our numerical results. Using this model and the solution approach, we study the impact of the investments of large-scale wind generation on long-term generation expansion planning.

2.3 Model Assumption

In this section, we discuss the settings and assumptions of our model. It is assumed that the power system consists of two types of generators: thermal and wind. The model considers a planning horizon of 4 stages, each of which spans 5 years. This model can be also applied to compute under other lengths of planning horizon by changing the values of the parameters without loss of generality. Since the temporal variability of load and wind power is mainly due to meteorological fluctuations of seasons and hours of the day [16], in our long-term model these factors have a relative short-term effect, and thus can use the average values.

Expansion planning models deal with the long-term investment problem, where long-term load growth and price trends are the main drivers for investment decisions. Given the size of the system and the multi-stage nature of the investment decisions, even simplified investment problems may become extremely complex and large-scale optimization problems if all operational constraints, as well as the stochastic, dynamic characteristics of the renewable generation and load are considered. Such operational details may result in a large bi-level (or tri-level) optimization problem, e.g., in [27]. In addition, it is often seen that results may not be that different from the case where some of operational constraints are simplified or ignored. For example, the commitment for thermal units can be absent. Besides, from the market perspective, similar considerations would hold for the modeling of all types of strategic behaviors of market competitors and the potential resulting equilibria as in [27]. Hence, in many of the previous studies, network effects which may be of less importance, have not been explicitly included (e.g., in the European context as discussed in [27] and [28]). Other cases without modeling network effects include short-term models [29, 30] and long-term models [16]. In addition, regardless of the network, the overall average electricity price has a negative relationship with the wind penetration [16]. Therefore, in this paper, since we focus on a new decision-dependent modeling approach, where the development of future uncertain prices depends on current investment decisions, we assume an aggregated level of operations and uncertain prices without explicitly modeling the network effects.

In this study, we assume the electricity network consists two types of generators units: thermal and wind, which is typical for the electricity network in the middle west. We also assume that the storage units are not considered in the network. This is because the energy storage units are mainly applied to deal with energy dispatch problems in the short-term market such as day-ahead unit commitment [31]. Our long-term planning horizon averages out the effect of storage units in the short-term.

The generation expansion requirement is determined by the future load demand level. In our model, levelized operation and maintenance cost c, levelized investment cost B, and unit investment cost b are considered as fixed and deterministic. The market price p^s, on the other hand, is assumed as the *endogenous uncertainty*. The market demand D is featured as having an overall increasing trend but affected by price variations.

2.3.1 *Planning Horizon and Scenario Tree Settings of the Uncertain Electricity Price*

Our study is aiming at long-term modeling where generation expansions are usually conducted via multiple steps/stages. We use a rooted scenario tree with multiple stages and branches to represent the planning horizon with uncertainty. There are two key features in the scenario tree: a time horizon divided into discrete stages, and each node (except the leaf) has several child nodes with different outcomes that represent the different realizations of uncertainty. For simplicity, these stages, $j \in \{1, 2, \ldots, J\}$, occur at evenly spaced increments of time. We denote the complete set of *nodes* of the scenario tree by $\mathcal{N}$, each of which represents a potential state of the market price of electricity. We also use the set $\mathcal{N}^- = \mathcal{N} \setminus \{1\}$ to represent all nodes starting from stage $j = 2$. For every node $n \in \mathcal{N}^-$ in stage j, we denote its unique *ancestor* node as $a(n)$ at stage $j - 1$. In contrast, $\mathcal{S}_n$ denotes the set of *successors* of node n in stage $j + 1$. Hence, we can use $\mathcal{S}_{a(n)}$ to denote the set of nodes that share the same ancestor node $a(n)$ in stage j. The root node is denoted as $n = 1$ which is in stage $j = 1$.

We assume that the electricity prices (at different time periods) are uncertain, and model them by using discrete random variables. At each ancestor node $a(n)$ (at time j), each node in the child node set $\mathcal{S}_{a(n)}$ is corresponding to an outcome/realization of the discrete random electricity price (at time $j + 1$). We use $p^{a(n)}$ and p^n to represent the prices in ancestor node $a(n)$ and node n, respectively. Then, δ^n, a prefixed parameter, is used to generate the outcome/realization of price at node n, through the equation,

$$p^n = p^{a(n)} \cdot (1 + \delta^n), \quad \forall n \in \mathcal{S}_{a(n)}. \tag{2.1}$$

For different nodes, δ^n is chosen differently. For example, in a binary tree, the two child nodes of $a(n)$ can have opposite values, e.g., $\pm 5\%$, to represent an increase and a decrease.

2.3.2 Modeling the Decision-Dependent Probability

While a power market is embracing more deregulation and competition, electricity prices and demands are directly influenced by the mix of the power generation capacity as in [32–34]. Wind generation's marginal cost (excluding its maintenance cost) is usually considered as zero. Hence penetration of wind power will undoubtedly decrease the electricity price. However, electricity prices are also considered as uncertain in many long-term expansion planning researches. It is important to link the price uncertainty with the expansion planning decisions. As discussed in Section **??**, one of the key features of our decision-dependent stochastic model is the decision-dependent probability distribution, which is modeled by a function of decision variables. In this paper, we discretize the electricity price in a known and fixed sample range.

In addition, we assume that the probabilities associated with given levels of electricity prices are not input parameters but are dependent on the investment decisions, as evidenced in the previous literature [13, 15, 16, 35, 36]. For example, researchers found that the average electricity price would decrease as the share of wind power in the generation portfolio increases. Moreover, a low-electricity-price scenario is more likely to happen if wind power's share is increasing. The opposite occurs for a high-electricity-price scenario. This is largely due to the fact that wind-power generation, compared to thermal generation, has a lower combined generation plus maintenance cost (c_i) for every magawatt hour of electricity it generates, even when we factor in the levelized investment cost (B_i) [1]. To model these findings, we propose a decision-dependent model to link probabilities of uncertain electricity price outcomes with investment decisions.

In our proposed model, we assume that the probability associated with any electricity price outcome (at node n) is a multivariate function of the possible future electricity price itself, generation portfolio (including both wind and thermal power capacity), combined generation and maintenance cost and levelized investment cost (per megawatt hour). In the scenario tree, every node (e.g., n representing a price outcome) is associated with a transition probability from its parent $a(n)$. As investment is a vital factor driving the electricity prices, for the decision-dependent uncertain electricity price, we assume that there is a positive relationship between the likelihood of a price outcome and its return or profitability on the investment. Motivated by [37], this probability is modeled as follows,

$$Prob^n = \frac{\sum_{i\in\{1,2\}} \frac{x_i^n(p^n - c_i - B_i)}{B_i(x_1^n + x_2^n)}}{\sum_{t\in\mathcal{S}_{a(n)}} \sum_{i\in\{1,2\}} \frac{x_i^t(p^n - c_i - B_i)}{B_i(x_1^t + x_2^t)}}, \ \forall n \in \mathcal{N}^-. \tag{2.2}$$

where $\mathcal{S}_{a(n)}$ is the set of nodes having the same parent node, $a(n)$. Based on the real-world data (see Table 2.4 from [1]), it is clear that $Prob^n \geq 0$. In addition, $\sum_{t\in\mathcal{S}_{a(n)}} Prob^t = 1$. As B_i is the levelized investment cost, we can use $\frac{(p^n - c_i - B_i)}{B_i}$ as a measure for the rate of the return on the investment of generation type i when price is p^n, and then $\sum_{i\in\{1,2\}} \frac{x_i^n}{x_1^n + x_2^n} \cdot \frac{p^n - c_i - B_i}{B_i}$ can be considered as a measure for the average rate of return on the total generation capacity. Equation (2.2) then defines the transition probability of a specific price outcome p^n (i.e., node n in the scenario tree) from its parent node $a(n)$. It is positively related to the return or profitability on the composition of the total generation asset. Note that Equation (2.2) presents the transition probability as a function of the market price p^n and the generation capacity x_i^n, $\forall i$. Unlike the traditional stochastic programming models with exogenous uncertainties where probability is fixed as a model's input parameters, the decision-dependent probability changes according to the investment decisions, making the model a decision-dependent stochastic programming model. In a nutshell, investment decisions affect the random price through influencing the probabilities of the outcomes while having a prefixed/known sample space of the price.

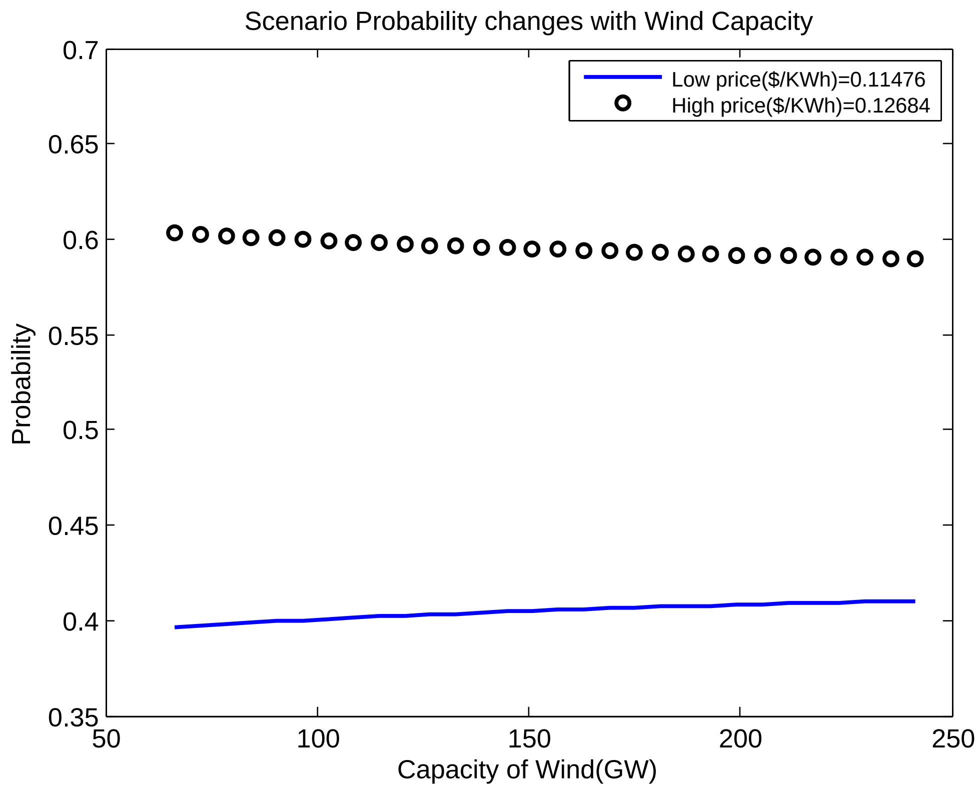

Figure 2.1: Price Probabilities vs. the Wind Capacity of the Power System

Figure 2.1 shows an example of the two-outcome probability curves that change in response to the installed wind capacity (i.e., the generation capacity mix as the thermal capacity is fixed) according to our model (2.2). The vertical axis represents the probability value, and the horizontal axis is the installed capacity of wind generation. The top and the bottom curves are representing the probability values under high-price and low-price outcomes respectively while perturbing the wind generation capacity from 50 GW to 250 GW. The thermal capacity is fixed at 306 GW. The two price outcomes used are 0.12684 \$/Kwh and 0.11476 \$/Kwh. All other data regarding levelized investment costs (B_i) and combined generation plus maintenance costs (c_i) for both types of generations are obtained from EIA data [1] as shown in Table 2.4. From the plot, we can observe that the probability is affected by both the price level and the wind capacity (i.e., its share of the total capacity as the thermal capacity is fixed). The two discrete outcome's probabilities are reflected by two separate curves on the plot. When the wind capacity is small, the high-price outcome has a higher probability. When the wind capacity is increasing, the high-price outcome's probability starts to decrease and the low-price outcome's probability starts to increase. In addition, the expected value of electricity price decreases while wind capacity penetration increases.

All of the above observations can be shown as corollaries of the following theorem.

Theorem 2.3.1. *In the two-outcome case with a high price outcome (P^H) and a low price outcome (P^L), the ratio between the corresponding probabilities, $Prob^H/Prob^L$, is a decreasing function of wind generation capacity, x_2.*

Proof. By plugging in the the formulas of probabilities of high and low prices defined in (2.2), we can have

$$\begin{aligned}\frac{Prob^H}{Prob^L} &= \frac{P^H - c_2 - B_2}{P^L - c_2 - B_2} \\ &+ \frac{\frac{x_1(c_1+B_1-c_2-B_2)(P^H-P^L)}{B_1}}{\frac{x_1(P^L-c_1-B_1)(P^L-c_2-B_2)}{B_1} + \frac{x_2(P^L-c_2-B_2)^2}{B_2}}\end{aligned}$$

Indices 1 and 2 are denoting thermal and wind generation respectively. Based on EIA data [1] (see Table 2.4), $c_1 + B_1 - c_2 - B_2 > 0$, i.e., that the thermal generation has a higher total sum of the levelized operations cost and the levelized investment cost. Also we know that $P^H - P^L > 0$, and then when x_2 increases, the ratio $\frac{Prob^H}{Prob^L}$ decreases. □

Corollary 2.3.1.1. *$Prob^H$ is a decreasing function of x_2.*

Proof. We know that $Prob^H + Prob^L = 1$. Hence $\frac{\partial Prob^H}{\partial x_2} = -\frac{\partial Prob^L}{\partial x_2}$. By Theorem 1, we know that $\frac{\partial \frac{Prob^H}{Prob^L}}{\partial x_2} < 0$. In addition,

$$\begin{aligned} \frac{\partial \frac{Prob^H}{Prob^L}}{\partial x_2} &= \frac{\frac{\partial Prob^H}{\partial x_2} Prob^L - \frac{\partial Prob^L}{\partial x_2} Prob^H}{(Prob^L)^2} \\ &= \frac{\frac{\partial Prob^H}{\partial x_2}}{(Prob^L)^2} \end{aligned}$$

Hence, $\frac{\partial Prob^H}{\partial x_2} < 0$. This also means that $Prob^L$ is an increasing function of x_2. □

Corollary 2.3.1.2. *The average electricity price is a decreasing function of wind generation capacity, i.e., x_2.*

Proof. Let AVP denote the expected price, and then $AVP = Prob^H P^H + Prob^L P^L$. The partial derivative with respect to x_2 is

$$\begin{aligned} \frac{\partial AVP}{\partial x_2} &= P^H \frac{\partial Prob^H}{\partial x_2} + P^L \frac{\partial Prob^L}{\partial x_2} \\ &= \frac{\partial Prob^H}{\partial x_2}(P^H - P^L) < 0 \end{aligned}$$

Hence, when x_2 increases, the expected price decreases. □

This price dependence on capacity is consistent with the results from other studies on real-world power systems. The long-term wind power investment study from [16] indicates that the average expected value of price decreases as wind farms are added. Similarly, the study from [36] also shows a decrease of average price due to the increasing wind power. Therefore, as mentioned in many studies, the price dependency on wind capacity is important for investors in evaluating the economic effects of power generation investments.

2.3.3 Generating the Market Demands as Inputs to the Model

Since the early seminal study of US electricity demand [38], electricity price and demand are founded closely linked. The relationship is depicted by the elasticity equations between electricity demand and price. Various research efforts have been taken to understand this relationship at both national and regional levels [39–41]. In this paper, we generate the demands as input parameters by assuming that the load/demand is affected by the electricity price variation. When price is increasing, the customer's desire to consume is lower, and therefore the load demand should be decreasing. Conversely, a large amount of demands could be stimulated by cheap prices. In this paper, we assume an elasticity model based on the well-known Tellis's econometric model [42]. The elasticity of demand to price is defined as $E = (\Delta D/\Delta p) \cdot (p/D)$, where p is the market price, E is the elasticity level, and D is the elastic demand. Based on this elasticity relation, we generate the demands in each node by using the following numerical expression of demand variation with respect to the corresponding price outcome (p^n), i.e.,

$$D^n = \frac{D^{a(n)}}{(p^{a(n)})^E} \cdot (p^n)^E \cdot (1 + e). \tag{2.3}$$

In addition, this equation constructs the connection between D^n and $D^{a(n)}$, which are the demand in node n and in its ancestor node $a(n)$, respectively. The parameter E represents the elasticity which is usually a negative value between -0.13 and -0.15 based on the study in [40], which covered the data of electricity price and demand relationship in US for more than two decades. Because E is chosen greater than -1 but negative, meaning the demand is not change much while price is varying, it is generally considered as inelastic (as opposed to the perfectly inelastic case, i.e., $E = 0$). The incremental level representing other factors (e.g., population growth, new electricity appliances) between demands in node n and its direct ancestor $a(n)$ is represented by e.

2.4 Mathematical Formulation

In this section, we propose a multi-stage decision dependent stochastic generation expansion investment model.

Table 2.1: Sets and Indices

$a(n)$	The *ancestor* of node n.
i	Index for types of generator: 1 for thermal, 2 for wind.
j	Index for stage, $j = 1, \ldots, J$.
l	Index for the binary variables introduced for linearization, $l = 1, \ldots, L$.
$\mathcal{N}$	The complete set all nodes of the scenario tree.
$\mathcal{N}^-$	The set of nodes excluding the one in the first stage.
$\mathcal{N}'$	The set of nodes excluding those in stage J.
n	Index for each node $n \in \mathcal{N}$.
$\mathcal{S}_n$	The *successor* set of node n in the next stage.

Table 2.2: Parameters

B_i	Levelized investment cost of thermal or wind $i \in \{1, 2\}$, in \$Billion/TWh.
b_i	Unit investment (overnight capital) cost of thermal or wind, $i \in \{1, 2\}$.
β_i	Capacity factor of thermal or wind $i \in \{1, 2\}$.
c_i	Levelized operation and maintenance cost $i \in \{1, 2\}$, in \$Billion/TWh.
D^n	The elastic demand in scenario tree node $\forall n \in \mathcal{N}$, in TWh.
δ^n	Price variation level at node n, $\forall n \in \mathcal{N}$.
E	Elasticity level of demand.
e	Incremental level between demands of two consecutive stages.
H	Number of hours in a planning stage, in Hour.
L	The number of binary variables used to represent the probability.
M_l	A large number to bound all continuous decisions.
p^n	Market price offered in scenario tree node n, $\forall n \in \mathcal{N}$, in \$Billion/TWh.
Rev^n	Total revenue in scenario tree node n, $\forall n \in \mathcal{N}$, in \$Billion.

Table 2.3: Variables

α_i^n	Future investment on capacity of thermal/wind in scenario tree node n, $\forall n \in \mathcal{N}$, in TW.
CO^n	Total operation cost in scenario tree node n, $\forall n \in \mathcal{N}$, in \$Billion.
ϵ^n	The error term when using binary variables to represent probability at node n.
g_i^n	Thermal or wind power production in scenario tree node n, $\forall n \in \mathcal{N}$, in TWh.
IC^n	Total investment cost in scenario tree node n, $\forall n \in \mathcal{N}$, in \$Billion.
$Prob^n$	Probability function of scenario tree node n, $\forall n \in \mathcal{N}$.
R^n	The total profitability of power generation asset composition in scenario tree node n.
SW^n	The profit in scenario tree node n, $\forall n \in \mathcal{N}$, in \$Billion.
θ^n	The variable to replace the bilinear term associated with the probability at node n.
x_i^n	Installed capacity for thermal/wind producer in scenario tree node n,$\forall n \in \mathcal{N}$, in TW.
z_l^n	The l^{th} binary variable used to represent the probability at node n.
ζ_l^n	The variable to replace the bilinear term representing the current profit.

The objective of power generation expansion planning is to maximize the total expected profit (based on the whole scenario tree), which is calculated as the difference between total revenue (Rev) and the total cost. The total revenue at each node n can be calculated by $Rev^n = p^n D^n$. The total cost consists of two parts: the total investment cost and the total operational cost (fuel costs plus maintenance costs). In our model, the investment costs (IC^n) are calculated on all nodes except the nodes associated with the last stage J. This is because the investment decisions are made to accommodate the future power system operations, and we assume an invested infrastructure at the current time period will be available starting from the next time period. For convenience, we use $\mathcal{N}'$ to denote the set of nodes having investment costs. The operating costs are calculated at each node of the scenario tree and include both generation costs (mainly thermal generators) and maintenance costs (both generation types). Hence, the objective function is a weighted sum of these revenues and costs, where the weights are simply the probabilities of the associated nodes in the scenario tree. Then we propose a multistage stochastic investment [**MSI**] model as follows,

$$\max \quad SW^1 \tag{2.4a}$$

$$\text{s.t.} \quad (2.2)$$

$$SW^{a(n)} = -IC^{a(n)} + \sum_{t \in \mathcal{S}_{a(n)}} Prob^t \cdot$$

$$\left(Rev^t - CO^t + SW^t\right),\ \forall n \in \mathcal{N}^- \tag{2.4b}$$

$$IC^n = \sum_{i\in\{1,2\}} b_i \alpha_i^n,\ \forall n \in \mathcal{N}' \tag{2.4c}$$

$$CO^n = \sum_{i\in\{1,2\}} c_i g_i^n,\ \forall n \in \mathcal{N}^- \tag{2.4d}$$

$$x_i^n = x_i^{a(n)} + \alpha_i^{a(n)},\ i \in \{1,2\},\ \forall n \in \mathcal{N}^- \tag{2.4e}$$

$$g_i^n \leq H\beta_i x_i^n,\ i \in \{1,2\},\ \forall n \in \mathcal{N}^- \tag{2.4f}$$

$$g_1^n + g_2^n = D^n,\ \forall n \in \mathcal{N}^- \tag{2.4g}$$

$$g_i^n, \alpha_i^n, w_i^n, Prob^n, x_i^n \geq 0,\ \forall i, n \in \mathcal{N} \tag{2.4h}$$

The decision variables α_i^n, x_i^n, and w_i^n respectively represent the future invested capacity, currently total, installed, cumulative capacity, and electricity generation of type i at node n. The cost parameter b_i, B_i ,c_i, represent the unit investment cost, levelized investment cost, and levelized operation and maintenance cost of generation type i, respectively. Note that x_i^0 is the initial installed capacities, which are given as parameters for both types of generators. The objective function (2.4a) has only one term: SW^1, which represents the total expected profit of the whole planning horizon, being calculated in a recursive way. Constraint (2.4b) defines the profit of the ancestor node $a(n)$ in stage $j-1$ that includes two terms: the investment cost $IC^{a(n)}$ and the expected total cost of node n's successors. The expected cost part consists of three terms: the operation cost CO^t, the revenue Rev^t, and the profit SW^t at node t, which is the immediate successor of node $a(n)$. Constraint (2.4c) defines the investment cost IC^n, which is determined by unit investment cost b_i and the new generation capacity α_i^n. The operational cost CO^n, given by constraint (2.4d), is determined by production level g_i^n. We assume that capacity expansion investment decisions made at time j will be ready to use at time $j+1$. Then the relation between current installed capacity and the future investment amount is given by constraint (2.4e). The power generation amount is also limited by the capacity factor in (2.4f). The capacity factors β_i represent the average ratio of currently installed capacity that can be utilized for generation. The power generation amount is enforced by (2.4g) to meet the load demand. According to Section 2.3.2, the decision-dependent price-capacity settings are included in (2.2) to capture the decision-dependent probability distributions.

2.5 Solution Approach

Since the constraints (2.4b) and (2.2) contain bilinear terms and fractional terms of decision variables, [**MSI**] is therefore a nonlinear optimization model. We first rewrite constraint (2.2) to be $Prob^n \cdot \sum_{t \in \mathcal{S}_{a(n)}} R^t = R^n$ to eliminate the fractional terms defining the probabilities, where $R^n = \sum_{i \in \{1,2\}} \frac{x_i^n(p^n - c_i - B_i)}{B_i(x_1^n + x_2^n)}$. In this way, the constraints (2.4b) and (2.2) both contain bilinear terms, $\sum_{n \in \mathcal{S}_{a(n)}} Prob^n \cdot (Rev^n - CO^n + SW^n)$ and $Prob^n \cdot \sum_{t \in \mathcal{S}_{a(n)}} R^t$. A bilinear term is the product of two decision variables and therefore makes the problem nonconvex and hence difficult to solve. As discussed in Section 2.1, linear-reformulation is widely used to solve nonconvex nonlinear optimization problems [21, 43–45]. We employ a quasi-exact method [26] to deal with the bilinear terms of our model. This method has a close link to Meyer and Floudas's [21] reformulation-linearization technique that reformulates the bilinear program (BLP) into mixed-integer linear programs (MILP). In both methods, the BLP is augmented with a set of binary variables. However, note that in our BLP model, the bilinear terms are very special which contain a continuous variable between 0 and 1, i.e., $Prob^n$. The quasi-exact method is specifically designed for this particular format. Unlike the reformulation-linearization technique that needs additional linear relaxation preprocessing, the quasi-exact approach uses a more straightforward approach that directly transforms the BLP to a series of bilinear products containing a binary variable and a continuous variable. Eventually, these products can be further linearized and therefore transformed to a series of mixed-integer linear programs. As the result, the constraints with bilinear terms can be formulated as a series of mixed-integer linear constraints.

This is because the quasi-exact approach is specifically designed to deal with bilinear terms that contains a fractional number between 0 and 1. We represent the variable $Prob^n$ via a series of binary variables. Eventually, the [**MSI**] model could be transformed to be a mixed-integer linear programming (MILP) problem, which can be solved conveniently by a state-of-the-art MILP solver.

2.5.1 *Solving the Bilinear Model through the Discretization-Linearization Procedure*

Given the definition of probability ($Prob^n$), it can only take a value between 0 and 1. In a modern computer system, any fractional number or variable x that is between 0 and 1 is represented by a series of binary variables $z_l \in \{0, 1\}$ [26], i.e., $x = \sum_{l=0}^{L} 2^{-l} z_l + \epsilon$ where L is the number of binary variables needed, and is related to the degree of accuracy. ϵ is the nonnegative error term. Its value is confined by L as $\epsilon < 2^{-L}$. Clearly, the more binary variables being used, the more accurate this approximation becomes.

Using the same approach, the variable $Prob^n$ can be discretized as follows,

$$Prob^n = \sum_{l=0}^{L} 2^{-l} z_l^n + \epsilon^n, \quad \forall n \in \mathcal{N}^- \tag{2.5}$$

Substituting $Prob^n$ in model [**MSI**] with the expression in (2.5), we have a new expression for constraints (2.4b) and (2.2):

$$\begin{aligned} SW^{a(n)} = -IC^{a(n)} + \sum_{t \in \mathcal{S}_{a(n)}} \left(\sum_{l=0}^{L} 2^{-l} z_l^t + \epsilon^t \right) \cdot \\ \left(Rev^t - CO^t + SW^t \right), \ \forall n \in \mathcal{N}^-, \end{aligned} \tag{2.6a}$$

$$\sum_{t \in \mathcal{S}_{a(n)}} R^t \cdot \left(\sum_{l=0}^{L} 2^{-l} z_l^n + \epsilon^n \right) = R^n, \ \forall n \in \mathcal{N}^- \tag{2.6b}$$

However, both z_l^n and ϵ^n are variables, and there still exist bilinear terms in (2.6a) and (2.6b).

These bilinear terms have the same format: a binary variable multiplied by a continuous variable. This type of bilinear terms can be easily linearized by introducing additional constraints and a big number, M_l [26]. For constraint (2.6a), we introduce a new variable ζ_l^n to replace the bilinear term:

$$\zeta_l^n = z_l^n \cdot \left(Rev^n - CO^n + SW^n\right), \; \forall n \in \mathcal{N}^-, l \tag{2.7}$$

Furthermore, we can replace the above equation with following equivalent constraints:

$$0 \leq \zeta_l^n \leq Rev^n - CO^n + SW^n, \; \forall n \in \mathcal{N}^-, \; l \tag{2.8a}$$

$$(Rev^n - CO^n + SW^n) - M_l(1 - z_l^n) \leq \zeta_l^n \leq M_l z_l^n, \forall n \in \mathcal{N}^-, \; l \tag{2.8b}$$

where M_l is a large number to bound the variables. For another term on the right side of constraint (2.6a), $\epsilon^n \cdot (Rev^n - CO^n + SW^n)$, there still exist bilinear terms with two continuous variables. However, this value is extremely small when enough binary variables (i.e., a large value for L) are used to represent the probability. As discussed in the previous part of this section, the range of the error term while representing the probability is: $0 \leq \epsilon^n < 2^{-L}$. Hence, we can introduce a new variable θ^n to represent the remaining bilinear term without losing accuracy by including the following constraint,

$$0 \leq \theta^n \leq 2^{-L} \cdot \left(Rev^n - CO^n + SW^n\right), \quad \forall n \in \mathcal{N}^-. \tag{2.9}$$

Combining equation (2.8) and equation (2.9), we can replace the nonlinear constraint (2.6a) with the following linear constraints,

$$SW^{a(n)} = -IC^{a(n)} + \sum_{t \in \mathcal{S}_{a(n)}} \left(\sum_{l=0}^{L} 2^{-l} \zeta_l^t + \theta^t \right), \; \forall n \in \mathcal{N}^-, \tag{2.10a}$$

$$0 \leq \theta^n \leq 2^{-L} \cdot \left(Rev^n - CO^n + SW^n\right), \; \forall n \in \mathcal{N}^-, \; l \tag{2.10b}$$

$$0 \leq \zeta_l^n \leq Rev^n - CO^n + SW^n,\ \forall n \in \mathcal{N}^-,\ l \tag{2.10c}$$

$$(Rev^n - CO^n + SW^n) - M_l(1 - z_l^n) \leq \zeta_l^n \leq M_l z_l^n, \forall n \in \mathcal{N}^-, \tag{2.10d}$$

Similarly, constraint (2.6b) can be converted as,

$$R^n = \sum_{l=0}^{L} 2^{-l}\eta_l^n + \sigma^n,\ \forall n \in \mathcal{N}^-, \tag{2.11a}$$

$$0 \leq \sigma^n \leq 2^{-L} \cdot \sum_{t \in \mathcal{S}_{a(n)}} R^t,\ \forall n \in \mathcal{N}^-,\ l \tag{2.11b}$$

$$0 \leq \eta_l^n \leq \sum_{t \in \mathcal{S}_{a(n)}} R^t,\ \forall n \in \mathcal{N}^-,\ l \tag{2.11c}$$

$$\sum_{t \in \mathcal{S}_{a(n)}} R^t - M_l(1 - z_l^n) \leq \eta_l^n \leq M_l z_l^n,\ \forall n \in \mathcal{N}^-, \tag{2.11d}$$

Because this quasi-exact solution process uses the error range of $[0,\ 2^{-L})$ to replace the error term ϵ^n, it is an approximation approach. Hence, the accuracy of our model depends on the number of binary variables used (L). So does the computational difficulty, but negatively. After reformulation, the number of constraints is equal to $4|\mathcal{N}^- \cdot |(2 + L) + |\mathcal{N}^{'}|$, where $|\mathcal{N}^-|$ and $|\mathcal{N}^{'}|$ represent the cardinality of set $\mathcal{N}^-$ and $\mathcal{N}^{'}$, respectively. Therefore, it is important to find a proper value of L to obtain high accuracy in a short computational time. This will be discussed in Section 2.6.2.

2.5.2 The Multi-stage Stochastic Mixed-Integer Linear Model

After the bilinear terms are discretized and therefore linearized, the bilinear constraints (2.4b) and (2.2) from [**MSI**] are replaced by the mixed-integer linear constraints (2.10) and (2.11). A multi-stage stochastic mixed-integer linear model [**MSMIL**] is therefore formulated as shown below:

$$\begin{aligned} \min \quad & SW^1 && \text{(2.12a)} \\ s.t. \quad & \text{(2.4c)} - \text{(2.4g)},\ \text{(2.4h)},\ \text{(2.10)},\ \text{(2.11)} && \text{(2.12b)} \\ & z_l^n \in \{0,1\},\ \forall n \in \mathcal{N}^-,\ l && \text{(2.12c)} \end{aligned}$$

2.6 Numerical Experiments

In this section, we present numerical experiments and analyze the results on generation expansion planning. Our model and algorithm are tested in a four-stage ($J = 4$) scenario tree. At first, Section 2.6.1 introduces the experimental settings as well as input data for our model. The fidelity of the quasi-exact approach is verified in Section 2.6.2 via a series of computational experiments to show the relation between the number of binary variables and the relative approximation error. Then, Section 2.6.3 tests the applicability of our quasi-exact approach via a series of comparisons with an existing commercial solver. Finally, the results of numerical experiments are discussed and analyzed in Section 2.6.4, 2.6.5, and 2.6.6. The computational model is programmed in C++ by calling the commercial MILP solver ILOG CPLEX 12.5. All experiments are implemented on a personal computer, which has quad Intel Core i7 processors with CPU at 3.40 GHz and a RAM space of 8GB.

2.6.1 Data Preparation

The input data for our model are acquired from US EIA [1], as shown in Table 2.4. We adopt the data of the conventional coal generator as the thermal generator, and the data of onshore wind farm as the wind generator. Compared to thermal generators, the wind generators have lower operation and investment costs. However, on the other hand, the capacity factor, which represents the average utilization of the total capacity, is lower for wind generator than that of thermal generators because of the nature of the unstable wind speed. Thus, with consideration of the capacity factor, the actual effective cost of investment and maintenance of wind is higher than that of thermal. The detailed data are shown in Table 2.4. As mentioned in Section 2.3.1, the market price is an uncertain parameter. The retail price at stage 1 (year 2015) is set at $\$0.15$/KWh. The variation level δ of price outcomes is adjusted according to experiment settings. As mentioned in Section 2.3.3, the load demand changes elastically with respect to the market price. The initial stage demand D_0 is set to be 12303.8 TWh. The number of hours H is set to be 43750 hours as we assume each stage spans 5 years. The elasticity index E, which reflects the correlation between demand and price, is accordingly set to be -0.15. The demand increasing level e is adjustable with different experiment settings.

Table 2.4: Input Parameters [1]

Parameter	Thermal	Wind	Unit
x^0	0.306	0.0604	TW
c	0.0345	0.013	Billion$ /TWh.
B	0.06	0.0641	Billion$ /TWh.
b	3292	2213	Billion$ /TW
β	0.85	0.30	N/A

2.6.2 *Accuracy of Model Approximation by MILP*

Through a series of computations, we study the accuracy of our quasi-exact linearized approximation approach with different values of binary variable (L). The input data of our calculation comes from Section 2.6.1, and we set the price variation level to be zero, that is, there is no difference between different price outcomes (i.e., nodes in the scenario tree). It is obvious that the probability $Prob^n$ of each node should be equal to each other, which is 0.5 for the two-node outcome. In this case, the [$\mathbf{MSI}$] model could be linearized by setting the variable $Prob^n$ to be a fixed value 0.5. Since $Prob^s$ is no longer a variable, this simplified [$\mathbf{MSI}$] model becomes a linear and deterministic model. Therefore, we can eliminate approximation, and solve the simplified [$\mathbf{MSI}$] model with a linear solver, providing a benchmark. Without the quasi-exact linearization process which brings in approximation, the optimization result of this deterministic model provides a benchmark for estimating the relative error of the quasi-exact linearized approximation approach.

Table 2.5 lists the error level, the relative error, the optimal profit, and computational time given to a series of numbers of binary variables (L). The error level is defined as 2^{-L}. The relative error is defined as the percentage difference between the profit SW of the deterministic model and the one from the quasi-exact approximation approach SW_L^A (using SW as the base), that is, $|SW_L^A - SW|/SW \times 100\%$.

We notice in Table 2.5 that as L increases, the computational time rises significantly, whereas the relative error decreases dramatically. When $L = 20$, the relative error is at the same level as the result of BARON. As a result, $L = 20$ is chosen as the initial approximation setting in the later case studies.

Table 2.5: Number of Binary Variables and Error

L	2^{-L}	Profit(10^9\$)	Relative Error	Time(s)
Deterministic Result:		4320.9214*	0	0.047
BARON's Result:		4320.9252	3.87×10^{-5}	136.91
5	3.13×10^{-2}	4599.6700	2.84×10^{-0}	0.27
10	9.77×10^{-4}	4329.4600	8.71×10^{-2}	0.61
15	3.05×10^{-5}	4321.1900	2.74×10^{-3}	5.43
20	9.54×10^{-7}	4320.9297	8.50×10^{-5}	15.21
25	2.98×10^{-8}	4320.9223	8.77×10^{-6}	25.17
30	9.31×10^{-10}	4320.9216	1.02×10^{-7}	92.32

$*$: Computed from simplified $[MSI]$ model with $Prob^s = 0.5$

2.6.3 Computation Comparison with Nonlinear Solver

To compare our quasi-exact approach with existing nonlinear solver, we embedded the bilinear [**MSI**] model in the global solver BARON. BARON is a state-of-the-art commercial software for solving nonconvex optimization problems to global optimality. We use BARON as a benchmark to test the applicability of our proposed approach. When solving an optimization problem, BARON reports an optimal solution (lower bound) and a upper bound. It declares global optimality when the corresponding optimality gap is less than a certain threshold.

In the following tests, we conducted the same numerical experiments in Section 2.6.4 by using both BARON and the proposed quasi-exact approach. The optimality gap of BARON is set at 10^{-6}. We report the relative difference between the optimal values from the quasi-exact approach and BARON. The computational time is also reported along with the results. Both solvers are implemented on the same personal computer.

Table 2.6: Computation comparison under different price variation levels

Price Uncertainty Level	Profit (10^9\$) Quasi-exact	BARON	Relative Difference (%)	Time (sec) Quasi-exact	BARON
0%	4320.930	4320.92	0.00%	15.21	247.75
2%	4327.731	4327.68	0.00%	52.59	475.24
4%	4354.652	4354.65	0.00%	25.12	159.07
6%	4401.900	4401.89	0.00%	45.98	159.00
8%	4467.879	4467.88	0.00%	85.53	207.21
10%	4553.001	4552.99	0.00%	161.29	238.01
12%	4658.699	4658.69	0.00%	156.00	603.87
14%	4785.292	4785.27	0.00%	175.31	608.28
16%	4929.447	4929.52	0.00%	172.88	996.42
18%	5093.699	5093.69	0.00%	126.74	493.31
20%	5278.769	5278.91	0.00%	284.35	588.53

In addition, we perform two series of tests. Firstly, we fix the demand incremental level, and change the price variation level and compare the computational differences between BARON and quasi-exact approach. The results is presented in Table 2.6. Table 2.7 presented the computational differences when we fix the price variation level but perturb the demand incremental level. From the results in Table 2.6 and Table 2.7, we notice that the relative difference in the optimal value between the proposed quasi-exact approach and BARON is always less than 0.01%. Hence, the optimality gaps are about at the same level for both solvers. This indicates that the quasi-exact method can provide equally accurate results as BARON, but with much less computational time on average.

Table 2.7: Computation comparison under different incremental levels for demands

Incremental Demand Level	Profit (10^9\$) Quasi-exact	Profit (10^9\$) BARON	Relative Difference (%)	Time (sec) Quasi-exact	Time (sec) BARON
0.0%	4484.35	4484.34	0.00%	144.30	125.56
0.2%	4498.00	4498.00	0.00%	102.17	304.03
0.4%	4511.57	4511.69	0.00%	95.72	313.03
0.6%	4525.44	4525.42	0.00%	101.92	454.07
0.8%	4539.19	4539.19	0.00%	111.45	279.68
1.0%	4553.00	4552.99	0.00%	48.33	238.01
1.2%	4566.85	4566.83	0.00%	374.16	503.07
1.4%	4580.71	4580.71	0.00%	94.44	360.57
1.6%	4592.87	4592.85	0.00%	221.90	477.34
1.8%	4604.58	4604.58	0.00%	121.11	606.18
2.0%	4616.34	4616.33	0.00%	334.81	521.48

In terms of computational time, the proposed quasi-exact approach finishes the computation in a shorter time in most of the cases. The quasi-exact method is able to acquire optimal results within 15 to 374 seconds under different price variation levels. On the other hand, the solution time of BARON varies greatly from 125 to 996 seconds for different cases. Especially, when the price variation level or increment demand level is getting larger, the computational time of BARON increases dramatically. This indicates that the quasi-exact method has a much more stable performance than BARON. In addition, it is notable that the computational time is not monotonically increasing while we are increasing the demand incremental level and the variation level of the price uncertainty. The quasi-exact model is a mixed integer linear program. With different data inputs (but the problem size is the same), the cutting planes from the solver might have different strengths and the branch-and-bound procedure might take different paths. Hence it is not predictable if high incremental level or price variation level means more computational time.

2.6.4 Analysis under Different Prices and Demands

The uncertain market price is one of the factors that affects the investment decision. In this case study, we try to understand the economic effects of market price under price variation levels from $\pm 0\%$ to $\pm 20\%$. Table 2.8 shows the results of this case study, including average market price, demand, profit, and wind penetration at each uncertainty price level. The average market price is calculated as the weighted average of market prices in all nodes. Wind capacity penetration is introduced to quantify the share of wind generators in the total power system's capacity as follows,

$$\text{Wind Capacity Penetration} = \frac{\text{Installed Wind Capacity}}{\text{Total Capacity}} \times 100\%$$

Table 2.8: Optimization result under different price variation levels

Price Uncertainty Level	Average Price (10^9\$/TWh)	Demand (TWh)	Profit (10^9\$)	Wind Penetration (%)
0%	0.150	49422.6	4320.93	16.150%
2%	0.152	49306.2	4327.73	16.194%
4%	0.155	49181.3	4354.65	16.159%
6%	0.158	49048.3	4401.90	16.158%
8%	0.161	48907.5	4467.88	16.134%
10%	0.165	48759.1	4553.00	16.094%
12%	0.169	48603.4	4658.70	16.053%
14%	0.173	48440.7	4785.29	16.014%
16%	0.177	48274.3	4927.45	15.982%
18%	0.182	48102.3	5093.70	15.941%
20%	0.187	47924.5	5278.77	15.940%

In Table 2.8, we see that as the variation level of the uncertain price increases, the average market price increases and demand decreases. It is because the demand is affected by the elastic relationship with the market price; thus, the demand shrinks as the price increases. We also notice that the wind penetration level decreases as the market price increases. In the investment problem of power systems, the more wind power we have, the lower the electricity price will be because of the price elasticity curve and the zero marginal cost of wind power. In this paper, prices (outcomes) are set as input parameters. Hence, when the average price increases, the wind power penetration is expected to decrease. This is in line with the previous literature [16, 32, 36]. Note that we show the data on average price, demand, and wind penetration. They are the average of all nodes in the scenario tree. However, the total installed capacity is not necessarily monotonically decreasing. This is because a larger decrease of price will lead to a larger increase of demand (based on the elasticity equation), and the investment in the parent node needs to cover the larger demand (from the low price-outcome node) in stochastic programming, causing the total capacity to increase. The profit is increasing as the average price is getting higher. The increment of the average price from $\$0.150/KWh$ to $\$0.187/KWh$ makes the profit increase by 22%. This increase in profit is attributed to two causes, i.e., the increasing revenue and the decreasing cost. On one hand, even with a small amount of demand decrease (3.03%), the large increase of market price (24%) appears to increase total revenue. On the other hand, lower demand results in a reduced cost in production and investment.

The demand is also an important factor that influences the generation expansion decisions, as shown in Table 2.9. To analyze the effect of the increasing demand on the power system, we conduct numerical experiments under different incremental levels of the demand while the price variation level is fixed at $\pm 10\%$. The results are shown in Table 2.9. It illustrates that wind penetration and the profit are correlated outputs: they both change according to different demands. As the demand increases, the wind penetration decreases and profit increases. When the demand increases, the needs for infrastructure expansion increase, which leads to more investments. The investment decisions tend to invest in less wind which has higher investment cost.

Table 2.9: Optimization result under different incremental demand levels

Demand Increment Level	Demand (TWh)	Profit (10^9\$)	Wind Penetration
0.0%	48039.2	4484.35	16.25%
0.2%	48182.4	4498.00	16.22%
0.4%	48326.0	4511.57	16.19%
0.6%	48470.0	4525.44	16.16%
0.8%	48614.3	4539.19	16.12%
1.0%	48759.1	4553.00	16.09%
1.2%	48904.2	4566.85	16.06%
1.4%	49049.7	4580.71	16.03%
1.6%	49195.6	4592.87	15.98%
1.8%	49341.9	4604.58	15.91%
2.0%	49488.5	4616.34	15.85%

2.6.5 Investment Decision Analysis

To illustrate the result of the [**MSMIL**] model graphically, we plot the optimization decisions in scenario trees, as shown in Figure 2.2. In Case 1, 2 and 3, the price's variation level is fixed at $\pm 10\%$. The incremental demand level is 1% in Case 1 and 2% in Case 2 and 3. Case 1 and 2 use the decision-dependent probability model from Section 2.3.2, the probability in Case 3 is set to be 0.5. The numbers above/below each branch represent the probability ($Prob^n$) of the outcomes. The two numbers within the parentheses represent the investment decisions (α_i^n) for thermal and wind generation, respectively. The numbers in each node represent the node number of the scenario tree. For the two child nodes of the same parent node, the market price in upper node is higher than the price in lower node.

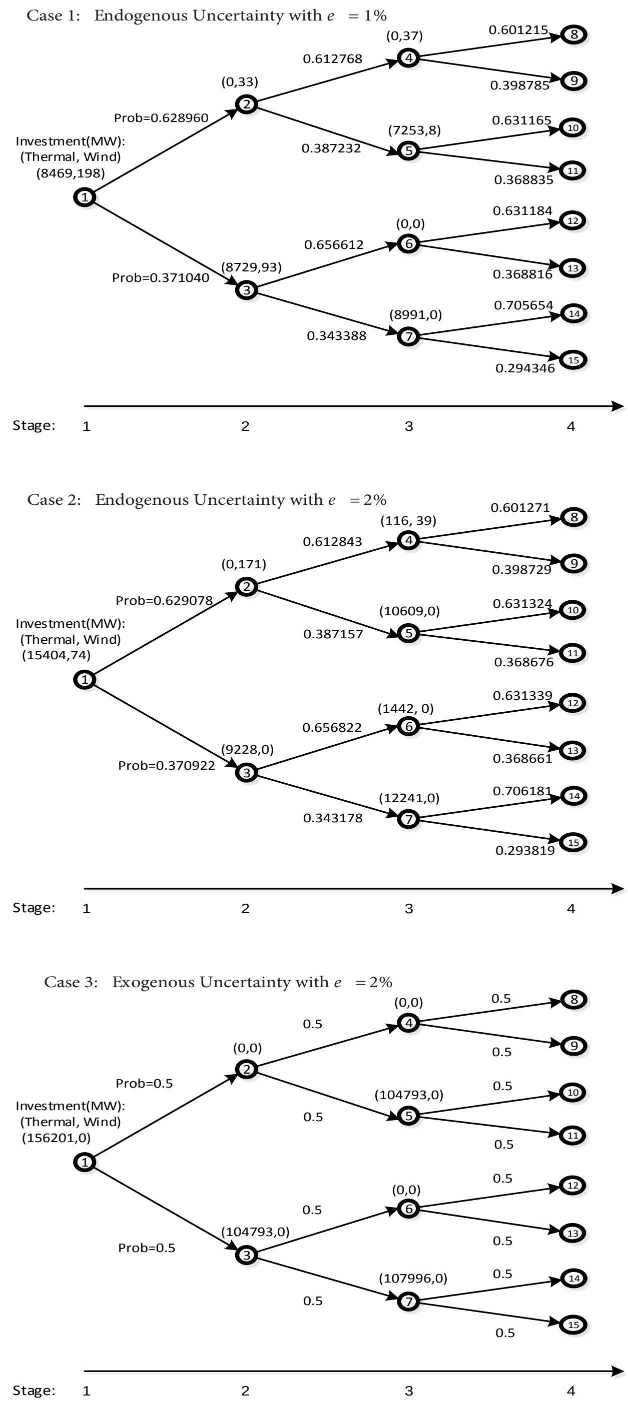

Figure 2.2: Investment Decisions and Probability at Each Outcome

Considering Case 1 and 2, from Section 2.3.2, we can see that the probability distribution is affected simultaneously by both the market price and investment decisions. In order to highlight the effect of investment decisions, the market price level is set the same between the first two cases in Figure 2.2. For each outcome, when the price level is given, the investment, on the other hand, plays an important role in determining the probability distributions. The following example shows how the investment decisions influence the decision-dependent probability distributions. While comparing the stage 2 (node 2,3) in Case 1 and 2, we note that, even though the price levels are the same in both cases, the probability distributions are different between Cases 1 and 2 ($\{0.628960, 0.371040\}$ vs. $\{0.629078, 0.370922\}$). This is because the investment in node 1 increases the wind capacity in stage 2 for both cases, but the amount of wind power investment in Case 1 is larger than Case 2 (198MW vs. 74MW). This makes the wind capacity in stage 2 of Case 1 larger than that in Case 2. Thus, this causes the probability distribution difference between the two cases. As a result, Figure 2.2 shows that investment decisions in stage 1 shift the decision-dependent probability distributions in stage 2.

From Section 2.6.1, we already know that the thermal generator has a low-cost investment advantage over wind. From the result in Case 3, we notice that without decision-dependent process, the traditional optimization decisions will focus all on the thermal generator for future investment. However, attributing to the decision dependent process, the results in both Case 1 and 2 show the investment decisions involve both thermal and wind generators.

2.6.6 Decision-Dependent Analysis

To examine the effectiveness of the decision-dependent approach, we introduce a term, the value of decision-dependent stochastic programming solution (VDDSS). It is extended from the concept of the value of stochastic programming solution (VSS). Unlike the VSS that compares a stochastic approach to a deterministic approach, the VDDSS evaluates the decision-dependent approach over the traditional stochastic approach (with exogenous uncertainty). To calculate the VDDSS, we first compute the optimal solution from a traditional stochastic model that uses the same input parameters and a prefixed probability distribution (e.g., uniformly distributed). Then, this solution is plugged into the decision-dependent formulation and the objective function value is then acquired. Finally, the VDDSS is calculated as the difference between the optimal objective value from decision-dependent approach and the objective value by using the traditional stochastic model solution in the decision-dependent model.

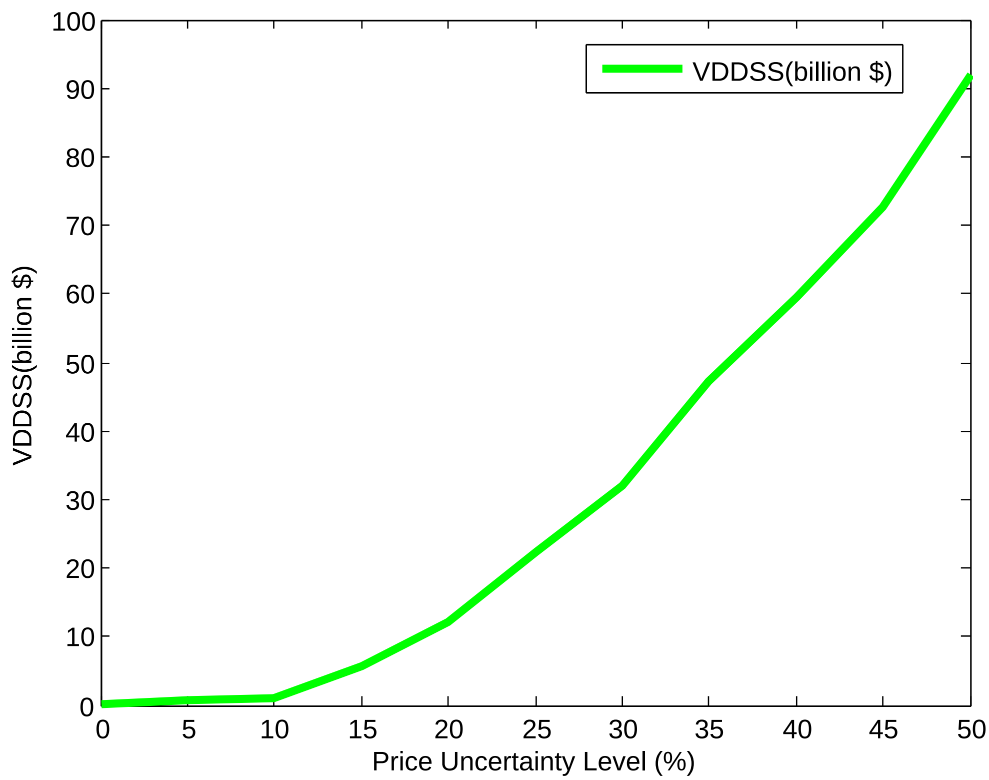

Figure 2.3: The Value of Decision-dependent Stochastic Programming Solution

Figure 2.3 shows that as price uncertainty changes from 0% to 50%, the VDDSS increases dramatically. When the price variation level is equal to zero, we observe that VDDSS is also equal to zero. This is because they both reduce to the same deterministic model. We observe that the VDDSS is greater than zero which indicates that the optimal solution from decision-dependent [**MSMIL**] formulation provides a larger profit than the one from the traditional stochastic programming approach. Moreover, as the price variation level increases (i.e., the difference of input parameters is greater between outcomes), the VDDSS is greater. Hence the decision-dependent approach outperforms the traditional stochastic programming approach especially when the price variation level is high. From these results, we can conclude that it is important to take into account the decision-dependent approach in evaluating the economics of long-term generation expansion planning where tremendous uncertainty exists and interplays with the investment decisions.

CHAPTER 3: BI-LEVEL DECISION DEPENDENT STOCHASTIC PROGRAMMING MODEL FOR POWER GENERATION INVESTMENT EXPANSION PLANNING

In recent years, the newly installed renewable generators especially the wind capacity has increased rapidly. The wind power has been participating in the electricity market in a large percentage. As a result, the investors would consider the possible investment decisions on both conventional and renewable power generators, which is called Generation Expansion Planning (GEP).

Apart from GEP problems, the expansion of Transmission planning (TEP) is also an important aspect in electricity system. However, in the electricity power system, the GEP and TEP problems are solved under very different frameworks. The GEP problem is usually solved by profit-oriented power investors with in a market environment. On the other hand, the TEP problem is generally solved by a central planner that determines the expansion plans that minimize the overall costs. Since in this chapter, our focus is to study the electricity system within the market environment, therefore, the TEP problems is out of the scope of this dissertation and is not considered in this chapter.

Within this context, in this chapter we consider a power investor that already owns a number of generator units and seeks at deciding both the optimal sizing, type and the optimal siting of the power generators units to be newly built or expanded within an electricity network. The objective of the electricity power investor is to maximize the expected profit from selling the electricity power production in the long term.

3.1 Introduction

The study of the investment for expansion in electricity generation problem has generally two different approaches: a centralized framework [46] and a market framework [36]. The centralized approach, like the one in chapter 2, determines the expansion plan based on the consideration of the whole electric system as a whole. Whereas, the market approach represents the electric market in which the producers participate and sell their power productions. In this chapter, we use the market approach to represent the perspective of a profit-oriented investor.

For this analysis, we formulate a bilevel multistage decision-dependent stochastic programming model considering thermal and wind power generator in long-term (10 to 20 years) future. Bilevel optimization is a branch of mathematical programming, it is a hierarchical relationship between two decision levels. It is originally come from the economic problem in the field of game theory [47]. This bilevel structure incorporates both long-term expansion planning and short-term generation and dispatch. It includes an upper-level stochastic expansion planning problem and a collection of lower-level problems that solves for optimal power flow (OPF).

The upper-level problem is formulated as a stochastic multi-stage long-term expansion planning problem. This problem seeks to determine the optimal investment plan for generators with the aim of maximizing the expected total profit. This investment plan considers multiple aspects including optimal siting, optimal sizing, and the optimal timing (investing stage) for both thermal and wind generators. The constructed generators will participate in the electricity market, offering its production at the price of local marginal price (LMP). Hence, the profit is achieved by selling the produced electricity to the market, and it depends on the market clearing prices with are obtained from lower-level problems.

The lower-level problem represents a DC optimal power flow (DCOPF) problem that seeks to find the minimum fuel cost by specifying the power generation and power flow within an electrical network [48]. Both existing and expanded power generators are considered to produce electricity to meet the demand in the network. The solution of the lower-level problems provide the market clearing price, which being used in the upper-level problem to compute the expected profit. The market clearing prices are considered as LMPs [49].

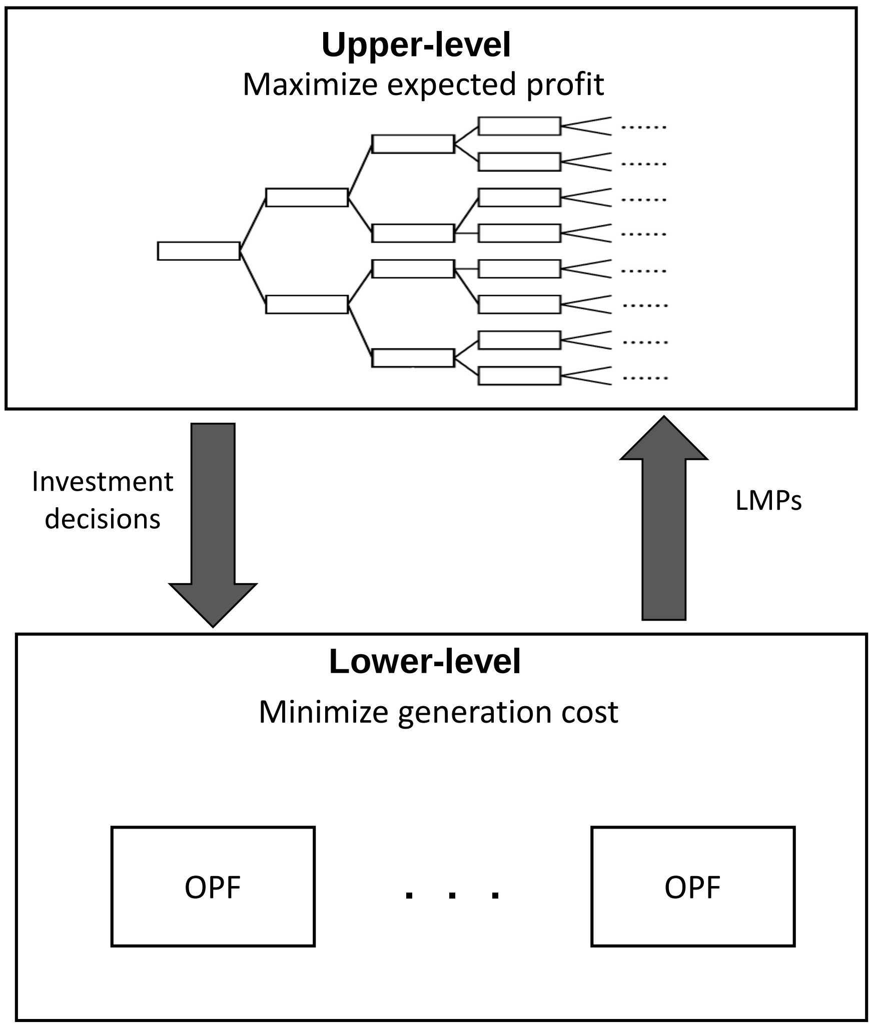

Figure 3.1: Bilevel structure

The interaction between upper-level and lower-lever problem is illustrated in Figure 3.1. The electricity power investor make expansion plans in the upper-level problem. The information of investment decisions are send to the lower-level problems, i.e. the electricity market. The lower-level problems calculate the optimal power generation and flows according to the investment decisions. The LMPs are provided within the solution of lower-level problems under different demand and wind intensity conditions. The upper-level problem calculates the maximum expected profit using the LMPs. Note that the upper-level and the lower-level problems are interconnected and must be solved jointly.

To deal with the uncertainty, similar to the approach in chapter 2, the upper-level problem is formulated as decision-dependent multistage stochastic programming. The stochastic formulation considers both *endogenous* uncertainty (i.e. unit investment cost) and *exogenous* uncertainty (i.e. demand and wind intensity). The probability distribution is considered as adjustable with decision variables.

In order to make our proposed model be solved more efficiently, several solution approaches are proposed to reduce the computation complexity of the original model. We first presents a linear transformation that uses duality relations to get rid of nonlinear terms in the revenue expression. Then, we take advantage of the property that the lower-level problems are continuous and linear, so they can be replaced by their Karush-Kuhn-Tucker (KKT) optimality conditions. Thus, the bilevel structure is transformed to a single level problem with the upper-level being constrained by the KKTs of all lower-level problems. This form of problem is called mathematical program with equilibrium constraints (MPEC). The MPEC is then converted to mixed-integer linear programming (MILP) by introducing binary variables. In order to improve the solution efficiency of the MILP, we employ Dantzig-wolfe decomposition [50] approach decompose the problem into a series of subproblems.

The rest of this chapter is arranged as follows. Section 3.2 reviews the methodologies from existing studies, and identifies the research gaps. Section 3.3 states model settings and assumptions. Then, the mathematical formulation is described in details in Section 3.4. The solution approaches are discussed in Section 3.5. The result of numerical case studies are presented in Section 3.6.

3.2 Literature Review

There are plenty of existing research study the generation investment decisions [9–11, 13, 14, 16]. The majority of them addresses the generation from convention energy sources. Recently, as the renewable energy growing rapidly in recent years, it has stimulated a number of studies that address the investment problem relates to wind power [11, 13, 14]. Kennedy [11] analysis the long-term cost and benefits that involves in wind power planning. Ivanova et.al [13] presents a multi-criteria approach for expansion planning considering wind power plants. Baringo et al. have conducted a series of studies that addresses on multiple topics of investment planning of wind power, including investment within market environment [36], investment on transmission and wind generators [51], investment with risk consideration [14], and strategic wind power investment with the aim of altering the market clearing prices [52].

The pricing of energy system has been widely studied. Baughman et al. [53, 54] set forth a comprehensive theory for real-time pricing in electricity. A lot of later studies adopt a simplified DC price theory that assumes the transmission network is DC. The study conducted by [55] presents a linear DCOPF market clearing framework that solves for local marginal price (LMP). The LMP is linked to the sensitivities of dual variables of the optimal DCOPF problem.

Bilevel programming problems have being widely applied in the application of energy field [30, 36, 56–60]. Gareces et al. [56] present a bilevel model for transmission expansion planning where the upper-level problem represents the target of transmission planner and market clearing problems are represented in lower-level. The study in [30] uses a bilevel model that has been transformed to mathematical program with equilibrium constraints to tackle the electricity price biding problems. Buijs et al. [58] proposed a bilevel optimization model to deal with transmission planning problem in a multilateral context. The proposed Pareto-planner maximizes overall welfare while guaranteeing that all zones can at least keep their initial level of welfare. Baringo and Conejo [36] deal with the profit-maximization problem of a wind power investor, where the clearing of the market under a variety of operating conditions. A stochastic bilevel optimization model is proposed, where upper-level represents wind investment decisions and lower-level represents the market clearing. Varies solution algorithms of bilevel programming problems have been proposed and adopted in several studies. Candler and Townsley [61] proposed an extreme-point approach for linear bilevel problems based on vertex enumeration. When the lower-level problem is convex and regular, the lower-level problem can be replaced by KKT conditions and therefore can be reformulated to be a single level problem. When the complementarity constraint is intrinsically combinatorial, it can be addressed by enumeration algorithms such as branch-and-bound [62]. Bialas et al. [63] also take advantage of KKT conditions in their solution algorithm: complementary pivoting. It is based on the reformulation of linear bilevel program using the KKT optimality conditions for the lower-level problem. There are also other solution methods being applied to solve bilevel programming problems, such as descent methods for convex bilevel program [64], penalty function methods for solving nonlinear bilevel prgramming problems [65], trust-region methods [66] and etc.

3.2.1 Research Gap

By summarizing previous approaches, we notice that following research gaps have not been covered. First, most of existing studies about electricity investment in the market environment only considers two-stage uncertain model. To the best of our knowledge, there are no multistage stochastic programming model explicitly representing the market environment (bilevel model).

Second, no decision dependent approach has been reported to address the electricity investment problem. Third, the converted MILP problems contains a large number of variables and constraints. There are very few studies have developed or applied advanced solution algorithms to the long-term electricity investment planning problem.

3.3 Model Setting and Assumption

In this study, we assume the electricity network consists two types of generators units: thermal and wind, which is typical for the electricity network in the middle west. We also assume that the storage units are not considered in the network. This is because the energy storage units are mainly applied to deal with energy dispatch problems in the short-term market such as day-ahead unit commitment [31]. Our long-term planning horizon averages out the effect of storage units in the short-term. The following sections discuss the uncertainty settings and decision-dependent probability settings.

3.3.1 Uncertainty Setting

In our model, we include both *exogenous* and *endogenous* uncertainties. The consideration of *exogenous* uncertainty includes uncertain demand and uncertain wind intensity. The *endogenous* uncertainty is reflected by the investment and maintenance cost of wind generator.

Because of the uncertain character of demand and wind intensity, it is important to properly model their uncertainty. In this study, we consider uncertain data given by Baringo [36], who has modeled the demand and wind uncertainty based on the load- and wind-duration curves from history data. According to this study, the demand and wind intensity varies greatly between different seasons. Therefore, we consider to divide each planning stage into four demand blocks, which correspond to four different seasons in a year. The uncertainty within each demand block is represented via exogenous uncertain scenarios. For example, each demand block may have two uncertain exogenous uncertain scenarios, high and low. Similarly, the wind intensity is also to be exogenous uncertain that associated with different wind intensity levels within each demand block.

For the sake of simplicity, the length of each block are set to be equal to each other, and both demand and wind are assumed to be normal distributed within each block. The uncertain demand level and wind intensity level are generated by the C++'s default random normal number generator function in the math.h library. The mean and standard deviation values for the uncertain demand and wind are acquired from IEEE test system.

We also incorporate the *endogenous* uncertainty for the cost of wind generators. This consideration comes from the fact that the wind energy is one of the most rapid growing energy source. The cost of wind generators is mainly contributed by the investment and the maintenance cost, that both of them has very big chance to vary in the future. The technology of wind turbine has experienced an immense growth during the last 30 years. Due to the recent undergoing important progress of power electronic device technology [67], both construction and maintenance cost would be likely to decrease in future. One the other hand, the new technologies are less mature and may have chance to contain defects which may led to the occurrence of high maintenance cost. Moreover, as the age of existing wind turbine increases, more maintenance is required for the components under intense and variable mechanical stress [68]. These potential issues, on the other hand, may also cause the increases of the wind generator's cost.

The uncertain investment and maintenance costs are characterized in different discrete levels in the uncertain scenarios. We denote a *node* in the scenario tree as n. For every node n, it has a unique *ancestor* node as $a(n)$. In contrast, $\mathcal{S}_n$ denotes the set of *successors* of node n. At each ancestor node $a(n)$, each node in the child node set $\mathcal{S}_{a(n)}$ is corresponding to an outcome/realization of the discrete random cost. We use $B^{a(n)}$ and B^n to represent the unit investment cost in ancestor node $a(n)$ and node n, respectively. Similarly, we use $m^{a(n)}$ and m^n to represent the unit maintenance cost in ancestor node $a(n)$ and node n, respectively. Then, δ^n, a prefixed parameter, is used to generate the outcome/realization of price at node n, through the equation,

$$B^n = B^{a(n)} \cdot (1 + \delta^n), \quad \forall n \in \mathcal{S}_{a(n)}. \tag{3.1a}$$

$$m^n = m^{a(n)} \cdot (1 + \delta^n), \quad \forall n \in \mathcal{S}_{a(n)}. \tag{3.1b}$$

For different nodes, δ^n is chosen differently. For example, in a binary tree, the two child nodes of $a(n)$ can have opposite values, e.g., $\pm 5\%$, to represent an increase and a decrease

3.3.2 Decision Dependent Probability

As discussed in Chapter 2, one of the key features of decision-dependent stochastic model is the decision-dependent probability distribution, which is modeled by a function of decision variables. We assume the probability associated with each outcome/realization varies according to the value of scale of economies (SCE) of the corresponding cost outcome.

3.3.2.1 The Scale of Economies (SCE) Model

The concept of economic scale in electricity market was first introduced in [69], it is described as a phenomenon that when the scale of production increases, the cost unit production would decrease. In [70], the author has derived that the combined cost to be a transcendental logarithmic function of key parameters and the measurement of the economic scale is the elasticity of the cost. The study in [71] introduced the variable of scale of economies (SCE). It is a variable between 0 and 1. The SCE measures the potential of cost decreasing in the electricity system. When SCE is close to

0, it means the system has "exploited" the potential of cost decreasing. On the other hand, if the SCE is close to 1, then the system should have large potential of reducing the cost.

In [71], the author presents a translog cost function as follows

$$\begin{aligned} \ln C_{ib} = \quad & \alpha_0 + \alpha_Y \ln g_{ib} + \frac{1}{2}\gamma_{YY}(\ln g_{ib})^2 + \sum_v \alpha_v \ln c_{ib,v} \\ & + \frac{1}{2}\sum_u \sum_v \gamma_{uv} \ln c_{ib,u} \ln c_{ib,v} + \sum_v \gamma_{Yv} \ln g_{ib} \ln c_{ib,v}] \end{aligned} \tag{3.2}$$

where C_{ib} and g_{ib} represent the total cost and amount of generation for generator i at bus b, respectively. $c_{ib,v}$ corresponds to the unit capital cost, unit operation and maintenance (OM) cost, and unit fuel cost with corresponding index v. α_0,α_Y, γ_{YY}, γ_{Yv} are coefficients from empirical data. The values of α_0,α_Y, γ_{YY}, γ_{Yv} are shown in Appendix.

Reference [71] also provides the expression of scale of economies (SCE). It is defined as a measurement that reflects the potential of the cost decreasing.

$$\begin{aligned} SCE_{ib} &= 1 - \frac{\partial \ln C_{ib}}{\partial \ln g_{ib}} \\ &= 1 - (\alpha_Y + \gamma_{YY} \ln g_{ib} + \sum_v \gamma_{Yv} \ln c_{ib,v}) \end{aligned} \tag{3.3}$$

In equation (3.3), the $c_{ib,v}$ includes unit generation, maintenance and capital costs. Therefore, this SCE evaluates a decreasing effect of the overall cost C_{ib} of the electricity system. For wind power specifically, this SCE depends on not only the amount of investment (i.e. capital cost) but also the amount of generation (i.e. maintenance cost).

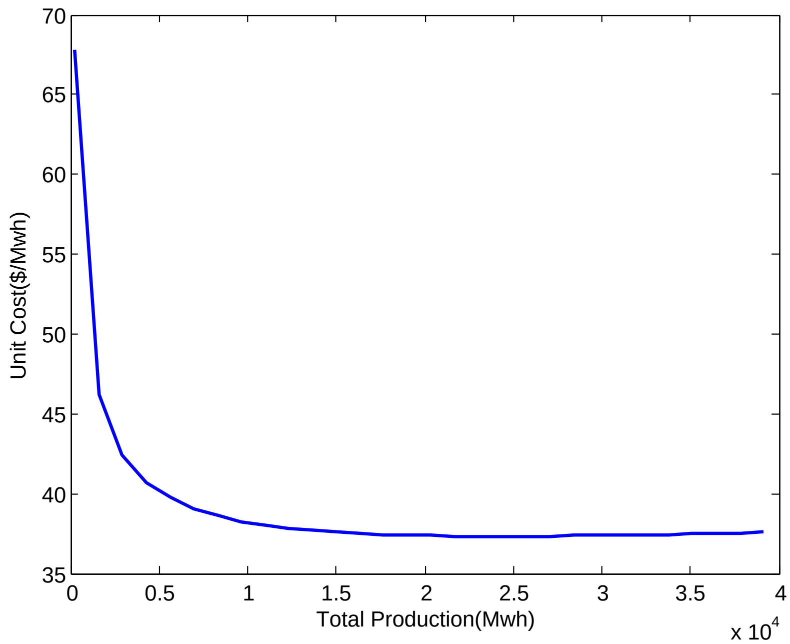

Figure 3.2: Unit Cost vs. the amount of generation

To further study the properties of cost and SCE, we have plot a cost-production curve using the data from IEEE reliability test system [72], shown in Figure 3.2. When the production level is low the unit cost decreases as in production growth. After production level passes a certain amount, i.e. g_0, the cost is no longer decreasing but starts to increase. In addition, the steepness of the cost curve is decreasing all the time as the production increases. The decrease of steepness indicates that the SCE value is decreasing as the production increases.

All of the above observations can be shown as following lemma and theorem.

Lemma 3.3.1. *The Scale of Economies SCE is a decreasing function in terms of generation g, regardless of the cost level c_v, i.e. $\frac{\partial SCE}{\partial g} < 0$.*

Proof. By plugging in the definition of SCE, we have

$$\frac{\partial SCE}{\partial g} = \frac{\partial(1 - \alpha_Y - \gamma_{YY}\ln g - \sum_v \gamma_{Yv}\ln c_v)}{\partial g} = -\frac{\gamma_{YY}}{g}.$$

Since $\gamma_{YY} > 0$ and $g \geq 0$, so $\frac{\partial SCE}{\partial g} < 0$, therefore Scale of Economies SCE is a decreasing function in terms of generation g. $\square$

Theorem 3.3.2. *The unit cost C_{unit} is a decreasing function when generation g is less than a certain amount of production g_0, i.e. $\frac{\partial C_{unit}}{\partial g} \leq 0$ when $g \leq g_0$.*

Proof. The unit cost C_{unit} is equal to total cost C divided by generation g, we have

$$\begin{aligned}
\frac{\partial C_{unit}}{\partial g} &= \frac{\partial(\frac{C}{g})}{\partial g} = \frac{\frac{\partial C}{\partial g}}{g} - \frac{C}{g^2} \\
&= \frac{g \cdot C}{g^2} \cdot (\frac{\alpha_Y}{g} + \frac{\gamma_{YY}\ln g}{g} + \frac{\sum_v \gamma_{Yv}\ln c_v)}{g}) - \frac{C}{g^2} \\
&= \frac{C}{g^2} \cdot (\alpha_Y + \gamma_{YY}\ln g + \sum_v \gamma_{Yv}\ln c_v - 1) \\
&= -\frac{C}{g^2} \cdot SCE
\end{aligned}$$

If $g \leq g_0$, then $SCE \geq 0$, C_{unit} is a decreasing function of g. According to the given data, we calculate that

- For wind generator, $g_0 = 2053Mwh$, i.e. when $g \leq 2053Mwh$, the unit cost C_{unit} is a decreasing function when generation g.
- For thermal generator, $g_0 = 30379Mwh$, i.e. when $g \leq 30379Mwh$, the unit cost C_{unit} is a decreasing function when generation g.

$\square$

3.3.2.2 The Probability

Considering the whole systems with different types of generator units, the total SCE should be considered as the sum of every bus b and every generator type i to be $\sum_{ib} SCE_{ib}$.

In order to distinguish the data from different scenario nodes, we add the index n to the equation (3.3) to be as follows,

$$\sum_{ib} SCE_{ib}^{n} = \sum_{ib} \left[1 - (\alpha_Y + \gamma_{YY} \ln g_{ib}^{n} + \sum_{v} \gamma_{Yv} \ln c_{ib,v}^{n}) \right] \tag{3.4}$$

where $c_{ib,v}^{n}$, as discussed in Section 3.3.1, are the realizations of uncertain costs (investment and maintenance) at node n. The variable g_{ib}^{n}, on the other hand, is the amount of generated electricity from generator type i at bus b of node n. This SCE value depends on both uncertain parameters (cost) and the decision variables (generation).

As we have acquired the scale of economies (SCE) for each node n in the scenario tree, we then can take advantage of discrete choice theory [73,74] to determine the probabilities of each outcome or node. The decision-dependent probability is formulated as follows,

$$Prob^{n} = \frac{\sum_{ib} SCE_{ib}^{n}}{\sum_{t \in \mathcal{S}_{a(n)}} \sum_{ib} SCE_{ib}^{t}}. \tag{3.5}$$

Equation (3.5) describes the probability of a particular cost realization. The probability $Prob^{n}$ is set as the ratio between the SCE at node n and the sum of the SCEs of all child nodes ($S_{a(n)}$). Note that the probability is a function of unit costs $c_{ib,v}^{n}$ and the amount of generation g_{ib}^{n}. Because g_{ib}^{n} is a decision variable, the values of g_{ib}^{n} are unknown before the optimization problem is solved.

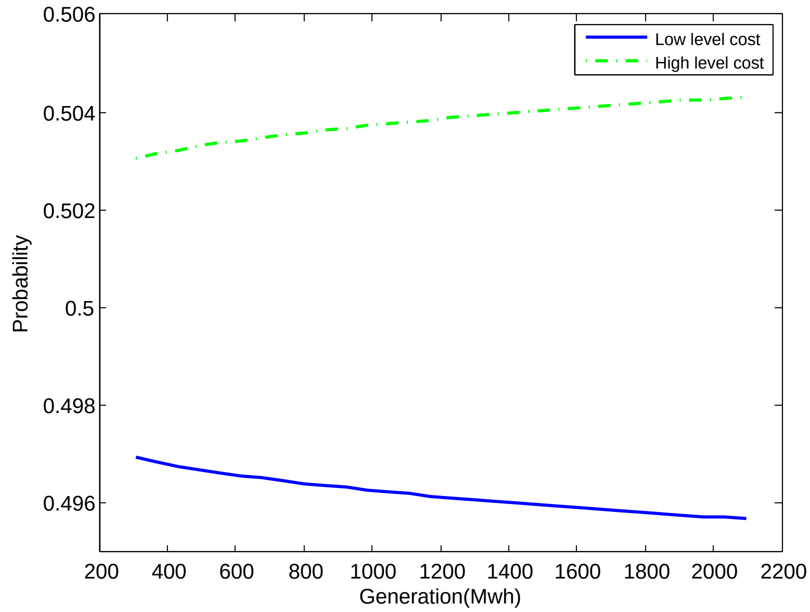

Figure 3.3: Probabilities vs. the amount of generation

Figure 3.3 shows an example of two-outcome probability curves that changes in response to the amount of generation. The vertical axis represents the probability value, and the horizontal axis is the generation amount. Each curve corresponds to the probability value in each cost outcome. From the plot, one can observe that the probability is affected by both the cost level and the generation amount. The two discrete outcome's probabilities are reflected by two separate curves on the plot. When the generation amount is small, the low-cost outcome has a higher probability. When the generation amount is increasing, the high-cost outcome's probability starts to increase and the low-cost outcome's probability starts to decrease.

All of the above observations can be shown as the following theorems and corollary.

Theorem 3.3.3. *In the two-outcome case with a high cost outcome (c_v^H) and a low cost outcome (c_v^L). The probability of high cost outcome $Prob^H$ is an increasing function of generation g, i.e.*

$\frac{\partial Prob^H}{\partial g} > 0$.

Proof. By plugging in the definition of $Prob^H$, we have

$$\begin{aligned}\frac{\partial Prob^H}{\partial g} &= \frac{\frac{\partial SCE^H}{\partial g}}{SCE^H + SCE^L} - \frac{SCE^H \cdot (\frac{\partial SCE^H}{\partial g} + \frac{\partial SCE^L}{\partial g})}{(SCE^H + SCE^L)^2} \\ &= -\frac{\gamma_{YY}}{g} \cdot \frac{SCE^L - SCE^H}{(SCE^H + SCE^L)^2} \\ &= -\frac{\gamma_{YY}}{g} \cdot \frac{\sum_v \gamma_{Yv}(\ln c_v^H - \ln c_v^L)}{(SCE^H + SCE^L)^2}\end{aligned}$$

Since $-\frac{\gamma_{YY}}{g} < 0$,$c_v^H > c_v^L$, and $\gamma_{Yv} < 0$, therefore $\frac{\partial Prob^H}{\partial g} > 0$. The probability of high cost outcome $Prob^H$ is an increasing function of generation g. □

Corollary 3.3.3.1. *In the two-outcome case with a high cost outcome (c_v^H) and a low cost outcome (c_v^L). The probability of low cost outcome $Prob^L$ is a decreasing function of generation g, i.e.* $\frac{\partial Prob^L}{\partial g} < 0$.

Proof. From the conclusion of Theorem (3.3.3), we have $\frac{\partial Prob^H}{\partial g} > 0$. Since

$$\begin{aligned}\frac{\partial Prob^L}{\partial g} &= \frac{\partial(1 - Prob^H)}{\partial g} \\ &= -\frac{\partial Prob^H}{\partial g} < 0\end{aligned}$$

Therefore the probability of low cost outcome $Prob^L$ is a decreasing function of generation g. □

3.4 Model Formulation

We formulate our problem in a bilevel framework to identify the optimal investment plan for generation expansion.

3.4.1 Upper-level Problem

In the upper-level formulation, we address the the long-term generation expansion problem to seek for maximizing the expected total profit. The decision variables are the investment decision of electricity generator units (both thermal and wind generators). The parameters and variables related to the upper-level model are summarized in Table 3.1 and 3.2.

Table 3.1: Upper-level Parameters and Indices

$t = 1, \dots, T$	The time periods (or stages) in the upper-level model
$\omega^t = 1 \dots, S_t$	Possible realizations of endogenous uncertainties (i.e., scenarios) at stage t
$k = 1, \dots, K_t$	Demand block (usually refer as season) at stage t
$\xi = 1, \dots, \Xi^k$	Possible realizations of endogenous uncertainties at season k.
$i = 1, 2$	Generator type. 1 for conventional, 2 for wind.
$b = 1, \dots,$ BNum	Bus number.
$B^t_{ib,\omega}$	Unit investment cost for generator i at bus b in scenario ω of stage t.
$c_{ib,\omega k}$	Unit production cost for generator i at bus b in scenario ω of stage t.
x^0_{ib}	Initial capacity of generator i at bus b.
$x^{\max}_{ib}$	Maximum allowed capacity for generator i at bus b.

Table 3.2: Upper-level Variables

$\alpha^t_{ib,\omega}$	Expansion decision for generator i at bus b in scenario ω of stage t.
$x^t_{ib,\omega}$	Capacity for generator i at bus b in scenario ω of stage t.
$g^t_{ib,k\omega\xi}$	Generation amount for generator i at bus b in scenario ω and ξ of stage t in demand block k.
$\lambda^t_{b,k\omega\xi}$	Local marginal price (LMP) at bus b in scenario ω and ξ of stage t in demand block k.
$IC^t_{\omega^t}$	Investment cost at stage t of scenario ω.
$REV^t_{\omega^t k\xi}$	Revenue in scenario ω and ξ of stage t in demand block k.

The total profit to be maximized is defined as the difference between the total revenue and the total investment cost. The formulation of the investment cost (denoted as IC) in time period t is given below,

$$IC^t_{\omega^t}(\alpha^t) = \sum_{ib} B^t_{ib,\omega}\alpha^t_{ib,\omega}, \ \forall, t, \omega \tag{3.6}$$

where $\alpha_{ib,\omega}^{t}$ is the investment decisions and $B_{ib,\omega}^{t}$ represents unit investment cost. In our model, the investment costs are calculated on all nodes except the nodes associated in the last stage T. This is because the investment decisions are made to accommodate the future power system operations, and we assume an invested infrastructure is only available in the next stage.

The total revenue is calculated as the product between the generation (denoted as $g_{ib,k\omega\xi}^{t}$) and unit price cost difference (denoted as $(\lambda_{b,k\omega\xi}^{t} - c_{ib,\omega k})$). The $\lambda_{b,k\omega\xi}^{t}$ is the local marginal price (LMP) that is solved in the lower-level model. $c_{ib,\omega k}$ is the unit production cost that includes both fuel cost and maintenance cost. The formulation of total revenue (denoted as REV) in the time period t is given below,

$$REV_{\omega^{t}k\xi}^{t} = \sum_{ib} (\lambda_{b,k\omega\xi}^{t} - c_{ib,\omega k}) \cdot g_{ib,k\omega\xi}^{t}, \ \forall \omega, t, \xi, k \tag{3.7}$$

The upper-level model varies in the time scale t, which represents each planning stage. In our model, we set the length of each stage to be 5 years. This model can be also applied to compute under other lengths of planning horizon by changing the values of parameters without loss of generality. Each planning horizon is divided into 4 demand block (denoted as k) to represents different demand and wind uncertain levels which has been discussed in Section 3.3.1.

The upper-level model is formulated as follows,

$$\begin{aligned}
\max_{\alpha} \mathcal{F}(\alpha;\omega) := & -IC^{0}(\alpha^{0}) + \\
& E_{\omega^{1}} \left\{ \max_{\alpha^{1}} \left[-IC_{\omega^{1}}^{1}(\alpha^{1}) + \sum_{k \in K_{1}} E_{\xi} \max_{\lambda_{k}^{1}, g_{k}^{1}} REV_{\omega^{1}}^{1,k}(\lambda_{k}^{1}, g_{k}^{1}) \right] + \cdots \right. \\
& \left. + E_{\omega^{T}} \left\{ \max_{\alpha^{T}} \left[-IC_{\omega^{T}}^{T}(\alpha^{T}) + \sum_{k \in K_{T}} E_{\xi} \max_{\lambda_{k}^{T}, g_{k}^{T}} REV_{\omega^{T}}^{T,k}(\lambda_{k}^{T}, g_{k}^{T}) \right] \right\} \cdots \right\}
\end{aligned} \tag{3.8a}$$

$s.t.$ Investment budget

$$\sum_{t=0}^{T} E_{\omega^t} IC^t(\alpha^t) \leq Bmax \tag{3.8b}$$

Investment capacity relations

$$x^t_{ib,\omega} = x^{t-1}_{ib,\omega} + \alpha^{t-1}_{ib,\omega},\ \forall i, b, t, \omega \tag{3.8c}$$

Capacity limitations

$$x^0_{ib} \leq x^t_{ib,\omega} \leq x^{\max}_{ib},\ \forall i, b, t, \omega \tag{3.8d}$$

Lower level connection

$$\lambda^t_{b,k\omega,\xi}, g^t_{ib,k\omega\xi} \in \arg\min\{\text{Lower-level Problem}\},\ \forall t, \forall k, \forall \omega, \tag{3.8e}$$

In the objective function (3.8a), the notation $E_{\omega^t}(\cdot)$ represents the expected value of different outcomes for *endogenous* uncertainty. Our stochastic model uses discrete probability distributions for each corresponded uncertain scenario. Thus, this notation of expected value $E_{\omega^t}(\cdot)$ is the same as $\sum_{n\in\mathcal{S}_{a(n)}} Prob^n \cdot (\cdot)$. The notation $E_\xi \max_{\lambda^1_k, g^1_k} REV^{1,k}_{\omega^1}(\lambda^1_k, g^1_k)$ represents the expected value is taken upon the realization of *exogenous* uncertainty ξ, which is reflected on the uncertain demand and wind intensity, as discussed in Section 3.3.1.

The constraints (3.8b) set the limit of total investment cost should not exceed the investment budget. Constraint (3.8c) states that the current available capacity ($x^t_{ib,\omega}$) is equal to the sum of capacity from last planning stage ($x^{t-1}_{ib,\omega}$) and the invested generators ($\alpha^{t-1}_{ib,\omega}$). The capacity of each generator is restricted in constraint (3.8d). Finally, constraints (3.8e) states that the values of variables $\lambda^t_{b,k\omega,\xi}$ and $g^t_{ib,k\omega\xi}$ are determined by lower-level problems.

3.4.2 Lower-level Problem

The lower-level problems are DC optimal power flow (DCOPF) that seeks to find the minimum fuel cost under different demand and wind intensity. The parameters and variables related to the lower-level model are summarized in Table 3.3 and 3.4. The model is formulated in Equation (3.9).

Table 3.3: Lower-level Parameters and Indices

$i = 1, 2$	Generator type. 1 for conventional, 2 for wind.
$b = 1, \ldots, \mathcal{N}_b$	Bus number.
$l = 1, \ldots, L$	Transmission line number.
$o(l)$	sending-end bus of line l.
$r(l)$	receiving-end bus of line l.
ref	Reference bus.
$b \setminus b : ref.$	bus except for the reference bus.
$c_{ib,\omega k}$	Unit production cost for generator i at bus b in scenario ω of stage t.
$d^t_{b,k\xi}$	Demand level at bus b in scenario ξ of stage t in season k.
H_k	Number of hours in demand block k.
S_l	Susceptance of line l.
$\beta_{i,k\xi}$	Utilization rate of generator i in demand block k of uncertain scenario ξ.

Table 3.4: Lower-level Variables

$g^t_{ib,k\omega\xi}$	Generation amount for newly expanded generators i at bus b in scenario ω and ξ of stage t in demand block k.
$p^t_{ib,k\omega\xi}$	Generation amount for existing generators i at bus b in scenario ω and ξ of stage t in demand block k.
$f^t_{l,k\omega\xi}$	Power flow via transmission line l in scenario ω and ξ of stage t in demand block k.
$\delta^t_{b,k\omega\xi}$	Phase angle at bus b in scenario ω and ξ of stage t in demand block k.

$$\min_{\forall t,k,\omega,\xi} \sum_{i,b} c_{ib,\omega}(g^t_{ib,k\omega\xi} + p^t_{ib,k\omega\xi}) \tag{3.9a}$$

$s.t.$ Power balance constraint

$$\sum_i (g^t_{ib,k\omega\xi} + p^t_{ib,k\omega\xi}) - \sum_{l|o(l)=b} f^t_{l,k\omega\xi} + \sum_{l|r(l)=b} f^t_{l,k\omega\xi} = d^t_{b,k\xi} \ : \ \lambda^t_{b,k\omega\xi} \quad \forall b \tag{3.9b}$$

Flow phase angle constraint

$$f^t_{l,k\omega\xi} = H_k S_l(\delta^t_{o(l),k\omega\xi} - \delta^t_{r(l),k\omega\xi}), \; : \;\; \phi^t_{l,k\omega\xi} \forall l \tag{3.9c}$$

Flow limitation constraint

$$-H_k f^{max}_l \leq f^t_{l,k\omega\xi} \leq H_k f^{max}_l \; : \; \phi^{min,t}_{l,k\omega\xi}, \; \phi^{max,t}_{l,k\omega\xi} \quad \forall l \tag{3.9d}$$

Generation limitation of new generators

$$0 \leq g^t_{ib,k\omega\xi} \leq H_k \beta_{i,k\xi} x^t_{ib,\omega} \; : \; \theta^{min,t}_{ib,k\omega\xi}, \theta^{max,t}_{ib,k\omega\xi} \quad \forall i, b \tag{3.9e}$$

Generation limitation of existing generators

$$0 \leq p^t_{ib,k\omega\xi} \leq H_k \beta_{i,k\xi} x^0_{ib} \; : \; \varphi^{min,t}_{ib,k\omega\xi}, \varphi^{max,t}_{ib,k\omega\xi} \quad \forall i, b \tag{3.9f}$$

Phase angle limitation

$$-\pi \leq \delta^t_{b,k\omega\xi} \leq \pi, \; : \; \eta^{min,t}_{b,k\omega}, \; \eta^{max,t}_{b,k\omega} \, \forall b \setminus b : ref. \tag{3.9g}$$

Phase angle for reference node

$$\delta^t_{b,k\omega\xi} = 0, \; : \; \chi^t_{b,k\omega} \, \forall b : ref. \tag{3.9h}$$

The objective function (3.9a) represents the minimization of generation cost of the existing and invested generators. Constraints (3.9b) enforce the supply-load balance at each node. The transmission flows are defined and limited in (3.9c) and (3.9d), respectively. The power productions of generation units are bounded in constraints (3.9e) and (3.9f). The parameters $\beta_{i,k\xi}$ represents the capacity factor of generators, which is a ratio of the capacity that can be utilized for generation. The wind intensity level is related to the capacity factor. Finally, constraints (3.9g) and (3.9h) enforce voltage angle be bounded at every node.

Because the electricity network is considered as a DC network, the LMPs can be considered as market clearing price [49]. The LMPs are the dual variables $\lambda^t_{b,k\omega\xi}$ for the constraints (3.9b).

3.5 Proposed Solution Approach

The proposed model in Section 3.4 is a multi-stage bilevel stochastic model with nonlinear constraints. In order to make this model efficiently solvable, we developed following approaches to reduce the computation complexity of our model.

3.5.1 Linear Transformation of Revenue Terms

In the upper-level problem's objective function (3.8a), the revenue term is defined in equation (3.7). It constitutes multiplication terms between two variables $\lambda^t_{b,k\omega\xi}$ and $g^t_{ib,k\omega\xi}$. An optimization problem that contains the product of two decision variables are call bilinear programming (BLP). Bilinear programming belongs to a class of nonconvex nonlinear optimization model. This nonlinear formulation will bring in great challenge of computation for the solution process.

In this section, we take advantage of KKT optimal conditions [75] of the lower-level problems (3.9) to derive the linear transformation of the revenue term.

The dual of the lower-level problem is shown as follows,

$$\max_{\forall k,\omega,\xi} \sum_b d^t_{b,k\xi}\lambda^t_{b,k\omega\xi} - \sum_l H_k f^{max}_l(\phi^{max,t}_{l,k\omega\xi} + \phi^{min,t}_{l,k\omega\xi})$$
$$- \sum_{ib} H_k\beta_{i,k\xi}(x^t_{ib}\theta^{max,t}_{ib,k\omega\xi} + x^0_{ib}\varphi^{max,t}_{ib,k\omega\xi}) - \sum_{b\setminus b:ref.} \pi(\eta^{min,t}_{b,k\omega} + \eta^{max,t}_{b,k\omega}) \tag{3.10a}$$

$$s.t.\ \lambda^t_{b,k\omega\xi} - \varphi^{max,t}_{ib,k\omega\xi} + \varphi^{min,t}_{ib,k\omega\xi} = c_{ib,\omega},\ \forall i,b \tag{3.10b}$$

$$\lambda^t_{b,k\omega\xi} - \theta^{max,t}_{ib,k\omega\xi} + \theta^{min,t}_{ib,k\omega\xi} = c_{ib,\omega},\ \forall i,b \tag{3.10c}$$

$$\lambda^t_{o(l),k\omega\xi} - \lambda^t_{r(l),k\omega\xi} - \phi^t_{l,k\omega\xi} + \phi^{max,t}_{l,k\omega\xi} - \phi^{min,t}_{l,k\omega\xi} = 0,\ \forall l \tag{3.10d}$$

$$- \sum_{l|o(l)=b} H_k S_l \phi^t_{l,k\omega\xi} + \sum_{k|r(k)=b} H_k S_l \phi^t_{l,k\omega\xi} + \eta^{max,t}_{b,k\omega} - \eta^{min,t}_{b,k\omega} = 0,\ \forall b\setminus b:ref. \tag{3.10e}$$

$$-\sum_{l|o(l)=b} H_k S_l \phi^t_{l,k\omega\xi} + \sum_{k|r(k)=b} H_k S_l \phi^t_{l,k\omega\xi} + \chi^t_{b,k\omega} = 0, \quad \forall b : ref. \tag{3.10f}$$

$$\phi^{max,t}_{l,k\omega\xi}, \phi^{min,t}_{l,k\omega\xi}, \theta^{max,t}_{ib,k\omega\xi}, \theta^{min,t}_{ib,k\omega\xi}, \varphi^{max,t}_{ib,k\omega\xi}, \varphi^{min,t}_{ib,k\omega\xi}, \eta^{min,t}_{b,k\omega}, \eta^{max,t}_{b,k\omega} \geq 0 \tag{3.10g}$$

$$\lambda^t_{b,k\omega\xi}, \phi^t_{l,k\omega\xi}, \chi^t_{b,k\omega} \text{ unrestricted} \tag{3.10h}$$

And we can also write out the complementary slackness relations as follows,

$$(H_k f^{max}_l + f^t_{l,k\omega\xi}) \cdot \phi^{min,t}_{l,k\omega\xi} = 0 \tag{3.11a}$$

$$(H_k f^{max}_l - f^t_{l,k\omega\xi}) \cdot \phi^{max,t}_{l,k\omega\xi} = 0 \tag{3.11b}$$

$$g^t_{ib,k\omega\xi} \cdot \theta^{min,t}_{ib,k\omega\xi} = 0 \tag{3.11c}$$

$$(H_k \beta_{i,k\xi} x^t_{ib} - g^t_{ib,k\omega\xi}) \cdot \theta^{max,t}_{ib,k\omega\xi} = 0 \tag{3.11d}$$

$$p^t_{ib,k\omega\xi} \cdot \varphi^{min,t}_{ib,k\omega\xi} = 0 \tag{3.11e}$$

$$(H_k \beta_{i,k\xi} x^0_{ib} - p^t_{ib,k\omega\xi}) \cdot \varphi^{max,t}_{ib,k\omega\xi} = 0 \tag{3.11f}$$

$$(\pi + \delta^t_{b,k\omega\xi}) \cdot \eta^{min,t}_{b,k\omega} = 0 \tag{3.11g}$$

$$(\pi - \delta^t_{b,k\omega\xi}) \cdot \eta^{max,t}_{b,k\omega} = 0 \tag{3.11h}$$

Next, we use the relations in (3.10) and (3.11) to replace the bilinear terms by a series of linear terms.

From dual constraints in (3.10c), we have:

$$\lambda^t_{b,k\omega\xi} - c_{ib,\omega} = \theta^{max,t}_{ib,k\omega\xi} - \theta^{min,t}_{ib,k\omega\xi} \tag{3.12}$$

Thus

$$\sum_{ib} (\lambda^t_{b,k\omega\xi} - c_{ib,\omega}) \cdot g^t_{ib,k\omega\xi} = \sum_{ib} (\theta^{max,t}_{ib,k\omega\xi} g^t_{ib,k\omega\xi} - \theta^{min,t}_{ib,k\omega\xi} g^t_{ib,k\omega\xi}) \tag{3.13}$$

Using complementary slackness equations of (3.11c) and (3.11d) to replace the right hand side of equation (3.13). We have the following,

$$\sum_{ib}(\lambda^t_{b,k\omega\xi} - c_{ib,\omega}) \cdot g^t_{ib,k\omega\xi} = \sum_{ib} H_k \beta_{i,k\xi} x^t_{ib} \cdot \theta^{max,t}_{ib,k\omega\xi} \tag{3.14}$$

Next, we take advantage of the condition that primal objective function (3.9a) and dual objective function (3.10a) must be equal, shown as follows,

$$\begin{aligned}\sum_{i,b} c_{ib,\omega}(g^t_{ib,k\omega\xi} + p^t_{ib,k\omega\xi}) \quad &= \quad \sum_b d^t_{b,k\xi}\lambda^t_{b,k\omega\xi} - \sum_l H_k f^{max}_l(\phi^{max,t}_{l,k\omega\xi} + \phi^{min,t}_{l,k\omega\xi}) \\ &\quad - \sum_{ib} H_k\beta_{i,k\xi} \cdot (x^t_{ib}\theta^{max,t}_{ib,k\omega\xi} + x^0_{ib}\varphi^{max,t}_{ib,k\omega\xi}) - \sum_{b\backslash b:ref.} \pi(\eta^{min,t}_{b,k\omega} + \eta^{max,t}_{b,k\omega})\end{aligned} \tag{3.15}$$

The RHS in (3.14) can be then replaced using the relations from the last step (3.15). Therefore, the nonlinear terms is finally replaced by a serise of linear terms.

$$\begin{aligned}\sum_{ib}(\lambda^t_{b,k\omega\xi} - c_{ib,\omega}) \cdot g^t_{ib,k\omega\xi} \quad &= \quad \sum_b d^t_{b,k\xi}\lambda^t_{b,k\omega\xi} - \sum_l H_k f^{max}_l(\phi^{max,t}_{l,k\omega\xi} + \phi^{min,t}_{l,k\omega\xi}) \\ &\quad - \sum_{i,b} c_{ib,\omega}(g^t_{ib,k\omega\xi} + p^t_{ib,k\omega\xi}) - \sum_{ib} H_k\beta_{i,k\xi}x^0_{ib}\varphi^{max,t}_{ib,k\omega\xi} \\ &\quad - \sum_{b\backslash b:ref.} \pi(\eta^{min,t}_{b,k\omega} + \eta^{max,t}_{b,k\omega})\end{aligned} \tag{3.16}$$

The LHS of (3.16) equal to the RHS of the nonlinear constraint (3.7). Therefore the revenue relation in (3.7) is replaced by:

$$\begin{aligned}REV^{t,k}_{\omega\xi} \quad &= \quad \sum_b d^t_{b,k\xi}\lambda^t_{b,k\omega\xi} - \sum_l H_k f^{max}_l(\phi^{max,t}_{l,k\omega\xi} + \phi^{min,t}_{l,k\omega\xi}) \\ &\quad - \sum_{i,b} c_{ib,\omega}(g^t_{ib,k\omega\xi} + p^t_{ib,k\omega\xi}) - \sum_{ib} H_k\beta_{i,k\xi}x^0_{ib}\varphi^{max,t}_{ib,k\omega\xi} \\ &\quad - \sum_{b\backslash b:ref.} \pi(\eta^{min,t}_{b,k\omega} + \eta^{max,t}_{b,k\omega}),\ \forall\omega, t, k, \xi\end{aligned} \tag{3.17}$$

3.5.2 *Transformation to MPEC and MILP*

The bilevel formulation requires the upper-level problem (3.8) and the lower-level problem (3.9) to be jointly solved. In this section, we replace the lower-level problem by its the KKT optimal conditions. Therefore, our bilevel problems is recast to be a single level optimization problem. This problem belongs to mathematical program with equilibrium constraints (MPEC). Its formulation is provided below:

$$\max_{\boldsymbol{\alpha}} \quad \text{(3.8a)} \tag{3.18a}$$

$$s.t. \quad \underbrace{\text{Constraints (3.8b)-(3.8e)}}_{\text{Upper-level constraints}} \tag{3.18b}$$

$$\{\underbrace{\text{Constraints (3.9b)-(3.9h)}}_{\text{Lower-level primal constraints}} \tag{3.18c}$$

$$\underbrace{\text{Constraints (3.10b)-(3.10h)}}_{\text{Lower-level dual constraints}} \tag{3.18d}$$

$$\underbrace{\text{Equations (3.11a)-(3.11h)}}_{\text{Complementary slackness relations}} \tag{3.18e}$$

$$\}, \ \forall t, k, \omega, \xi \tag{3.18f}$$

The complementarity constraints (3.11a)-(3.11h) can be reformulated through exact equivalent mixed-integer linear equations using Fortuny-Amat transformation [76]. All of the complementarity constraints have the form $\alpha \cdot \gamma = 0$. It can be linearized to two equivalent constraints: $\alpha \leq M \cdot u$ and $\gamma \leq M(1-u)$, where M is a sufficiently large constant and u is binary variable. In this way, the $MPEC$ is converted to a MILP.

Finally, the generation investment model is can be formulated as an MILP as below,

$$\max_{\boldsymbol{\alpha}} \quad (3.8a) \tag{3.19a}$$

subject to

$$\text{Constraints (3.8b)-(3.8e)} \tag{3.19b}$$

$$\{$$

$$\text{Constraints (3.9b)-(3.9h)} \tag{3.19c}$$

$$\text{Constraints (3.10b)-(3.10h)} \tag{3.19d}$$

$$\phi^{min,t}_{l,k\omega\xi} \leq M \cdot u^{\phi^{min,t}}_{l,k\omega\xi}, \quad \forall l \tag{3.19e}$$

$$\phi^{min,t}_{l,k\omega\xi} \leq M \cdot u^{\phi^{max,t}}_{l,k\omega\xi}, \quad \forall l \tag{3.19f}$$

$$\theta^{min,t}_{ib,k\omega\xi} \leq M \cdot u^{\theta^{min,t}}_{ib,k\omega\xi}, \quad \forall i, b \tag{3.19g}$$

$$\theta^{max,t}_{ib,k\omega\xi} \leq M \cdot u^{\theta^{max,t}}_{ib,k\omega\xi}, \quad \forall i, b \tag{3.19h}$$

$$\varphi^{min,t}_{ib,k\omega\xi} \leq M \cdot u^{\varphi^{min,t}}_{ib,k\omega\xi}, \quad \forall i, b \tag{3.19i}$$

$$\varphi^{max,t}_{ib,k\omega\xi} \leq M \cdot u^{\varphi^{max,t}}_{ib,k\omega\xi}, \quad \forall i, b \tag{3.19j}$$

$$\eta^{min,t}_{b,k\omega} \leq M \cdot u^{\eta^{min,t}}_{b,k\omega}, \quad \forall b \setminus b : ref. \tag{3.19k}$$

$$\eta^{max,t}_{b,k\omega} \leq M \cdot u^{\eta^{max,t}}_{b,k\omega}, \quad \forall b \setminus b : ref. \tag{3.19l}$$

$$H_k f^{max}_l + f^t_{l,k\omega\xi} \leq M \cdot (1 - u^{\phi^{min,t}}_{l,k\omega\xi}), \quad \forall l \tag{3.19m}$$

$$H_k f^{max}_l - f^t_{l,k\omega\xi} \leq M \cdot (1 - u^{\phi^{max,t}}_{l,k\omega\xi}), \quad \forall l \tag{3.19n}$$

$$g^t_{ib,k\omega\xi} \leq M \cdot (1 - u^{\theta^{min,t}}_{ib,k\omega\xi}), \quad \forall i, b \tag{3.19o}$$

$$H_k \beta_{i,k\xi} x^t_{ib} - g^t_{ib,k\omega\xi} \leq M \cdot (1 - u^{\theta^{max,t}}_{ib,k\omega\xi}), \quad \forall i, b \tag{3.19p}$$

$$p^t_{ib,k\omega\xi} \leq M \cdot (1 - u^{\varphi^{min,t}}_{ib,k\omega\xi}), \quad \forall i, b \tag{3.19q}$$

$$H_k \beta_{i,k\xi} x^0_{ib} - p^t_{ib,k\omega\xi} \leq M \cdot (1 - u^{\varphi^{min,t}}_{ib,k\omega\xi}), \quad \forall i, b \tag{3.19r}$$

$$\pi + \delta^t_{b,k\omega\xi} \leq M \cdot (1 - u^{\eta^{min,t}}_{b,k\omega}), \quad \forall b \setminus b : ref. \tag{3.19s}$$

$$\pi - \delta^t_{b,k\omega\xi} \leq M \cdot (1 - u^{\eta^{max,t}}_{b,k\omega}), \quad \forall b \setminus b : ref. \tag{3.19t}$$

$$u^{\phi^{min,t}}_{l,k\omega\xi}, u^{\phi^{max,t}}_{l,k\omega\xi}, u^{\theta^{min,t}}_{ib,k\omega\xi}, u^{\theta^{max,t}}_{ib,k\omega\xi}, u^{\varphi^{min,t}}_{ib,k\omega\xi}, u^{\varphi^{max,t}}_{ib,k\omega\xi}, u^{\eta^{min,t}}_{b,k\omega}, u^{\eta^{max,t}}_{b,k\omega}, \in \{0, 1\}, \tag{3.19u}$$

$$\}, \; \forall t, k, \omega, \xi \tag{3.19v}$$

where M is a sufficient large enough constant.

3.5.3 Linearization Heuristics for Decision-dependent Probability

From Section 3.3.2.2, we know that our probability $Prob$ is a function of decision variables g. This will introduce nonlinear terms to the objective function. Thus we employ an iterative heuristic method to avoid this nonlinear formulation.

The process is as follows. We first acquire an initial solution $\mathbf{g}^{\mathbf{ini}}$ to compute the value of probability $Prob(\mathbf{g}^{\mathbf{ini}})$. Then, we replace the decision-dependent probability $Prob(\mathbf{g})$ by $Prob(\mathbf{g}^{\mathbf{ini}})$, which is a fixed value to get rid of nonlinear term. After this step, the linear model is solved by MILP solver with the optimal solution $\hat{\mathbf{g}}_1$ and objective value $\hat{Z}_1$. In the next step, the decision-dependent probability is replaced by $Prob(\hat{\mathbf{g}})$. The model is then solved with optimal solution $\hat{\mathbf{g}}_2$ and objective value $\hat{Z}_2$. The heuristic process is then solved iteratively until the stopping criteria is reached at iteration i as $\frac{|\hat{Z}_i - \hat{Z}_{i-1}|}{\hat{Z}_i} < \epsilon$.

3.5.4 Dantzig-Wolfe Decomposition Approach

The formulation in (3.19) is a multistage stochastic mixed integer program with a large number of constraints and variables. Although we have taken advantage of state-of-art MILP solver to solve the problem, but the computational time is still very long. To identify the underlying computation complexity, we compare the computation time between deterministic model and the stochastic model, shown in Table 3.5. The computation time of deterministic model is much shorter than the time of stochastic model. This may imply that the computation complexity is embedded with the stochastic structure.

Table 3.5: Deterministic vs. Stochastic Computation Time

Bus	Computation time (sec)	
	Deterministic	Stochastic
3	2.84	22.78
30	49.52	64558.26
57	18.52	(4.34%)*
118	73.16	(4.15%)*

$*$: Exceeded time limit of 80000 seconds.

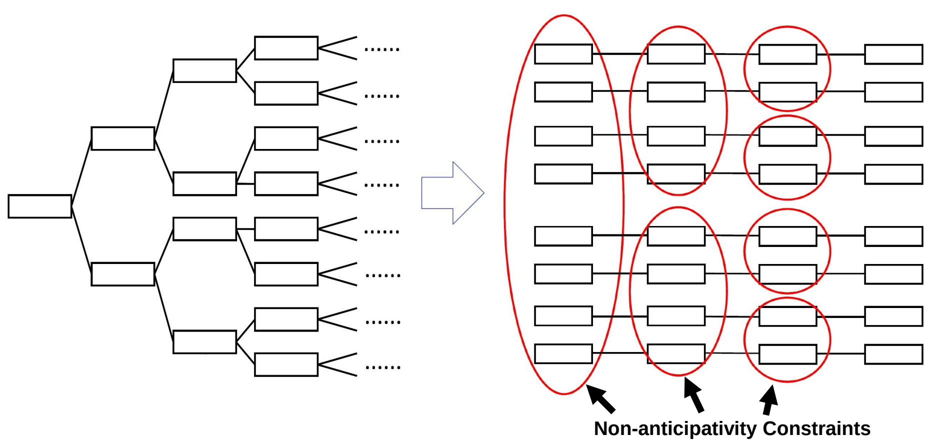

Figure 3.4: Scenario splitting

We first transform the nodal based formulation of the stochastic problem into scenario based formulation, i.e. the formulation is based on unique paths from the root node to the leave nodes. The scenarios are connected via non-anticipativity constraints shown in Figure 3.4. The structure of the scenario based constraints is illustrated in Figure 3.5.

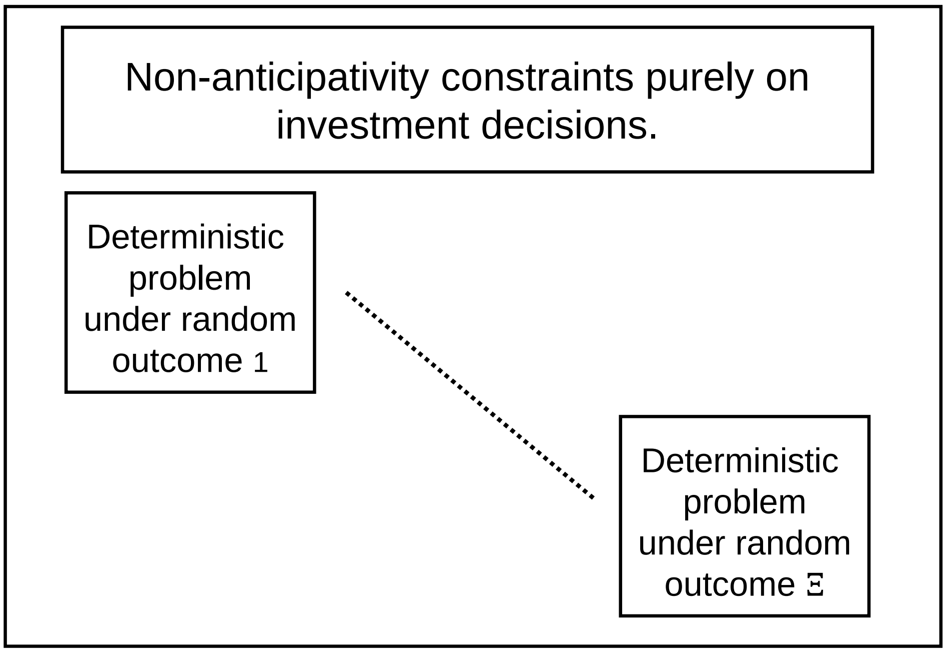

Figure 3.5: Problem Structure

The simplified reformulation is shown as follows,

$$\max \sum_{s\in\mathcal{S}} Prob_s \sum_{t\in\mathcal{T}} [c_s x_{st} + E_\xi d_s(\xi) y_{st}] \tag{3.20a}$$

$$s.t.\ x_{st} = x_{n(s,t)}, \quad \forall s \in \mathcal{S}, t \in \mathcal{T}, n \in \mathcal{N} \tag{3.20b}$$

$$\{ \tag{3.20c}$$

Constraints (3.19b)-(3.19u), (3.20d)

$$\}\forall s \in \mathcal{S}, t \in \mathcal{T}, \xi \in \Xi \tag{3.20e}$$

where $\mathcal{N}$ is the set of nodes of the nodal based scenario tree, $\mathcal{S}$ represents all the scenarios, and $\mathcal{T}$ at stages. x_{st} represents the here-and-now variables, specifically investment decisions, and y_{st} represents wait-and-see variables, specifically operation related variables. Constraints (3.20b) are non-anticipativity constraints that enforce all the variables that belonging to the same node should be equal. The non-anticipativity constraints bound different scenarios to be one integrated problem.

In Dantzig-Wolfe decomposition method, the stochastic structure is decomposed into a master problem and a set of subproblems. Each subproblem represents a scenario in the scenario tree. The master problem incorporates the solutions from each subproblem to acquire the final optimal solution.

$$\text{[RMP]:} \quad \max \sum_{s\in\mathcal{S}} Prob_s \cdot \left[-\sum_{t\in\mathcal{T}} IC_{st} + \sum_{j\in\mathcal{F}_s} \rho_s^j \sum_{t\in\mathcal{T}} \sum_{k} E_\xi(R\hat{E}V^j_{st,k\xi}) \right] \tag{3.21a}$$

$$s.t. \sum_{j\in\mathcal{F}_s} \rho_{st}^j \cdot \hat{z}^j_{st,ib} \leq \alpha_{n(s,t),ib}, \forall s \in \mathcal{S}, t \in \mathcal{T}, n \in \mathcal{N}, i, b, \ : \ \pi_{st,ib} \tag{3.21b}$$

$$\sum_{j\in\mathcal{F}_s} \rho_s^j = 1, \ \forall s \in \mathcal{S}, \ : \ \pi_s^0 \tag{3.21c}$$

$$\rho_s^j, \alpha_{n,ib} \geq 0, \forall n, j \in \mathcal{F}_s \tag{3.21d}$$

where $\mathcal{F}_s$ is the set of all feasible investment solutions for scenario s. $\pi_{st,ib}$ and π_s^0 are dual variables. The [RMP] calculates the overall optimal investment solution $\alpha_{n,ib}$ from the convex combination of the feasible investment solutions $\hat{z}^j_{st,ib}$ that are acquired by solving subproblems for each scenario s.

$$\text{[SP}_s\text{]:} \quad \max_{\forall s\in\mathcal{S}} \quad Prob_s \cdot \left[\sum_{t\in\mathcal{T}} \left(\sum_{k} E_\xi(REV_{st,k\xi}) - \sum_{ib} \hat{\pi}_{st,ib} z_{st,ib} \right) - \hat{\pi}_n^0 \right] \tag{3.22a}$$

$$s.t. \ \{$$

$$\text{(3.19b)} - \text{(3.19u)}, \tag{3.22b}$$

$$\}, \forall t, \forall k, \forall \omega, \forall \xi$$

The subproblems aim at evaluate the reduced costs of extreme points of master problem. The most preferable reduced costs are determined to enter the basis. The master and sub problems are solved iteratively. At each iteration the [RMP] is updated by adding columns until the convergence is achieved.

The solution algorithm including Dantzig-Wolfe decomposition and linearization heuristics for decision-dependent probability, is sumarized as follows,

Algorithm 1 Solution algorithm for electricity investment planning

Initialize: $i = 1$, $\hat{\mathbf{g}}_1 = \mathbf{g}^{ini}$.
while $|\hat{Z}_i - \hat{Z}_{i-1}|/\hat{Z}_i > \epsilon_1$ **do**
    Compute $Prob(\hat{\mathbf{g}}_i)$
    Initialize the [RMP] with UB=$+\infty$, LB=$-\infty$,$j = 1$, $\hat{\mathbf{x}}_1 = \mathbf{x}_0$.
    while (UB-LB)/LB$> \epsilon_2$ **do**
        Solve the [RMP] and update LB to be its optimal value Z_{RMP}.
        Update the optimal investment decision $\mathbf{x}^* = \hat{\mathbf{x}}_j$.
        Solve the [SP$_s$], record their optimal values Z_{SP_s} $\forall s \in \mathcal{S}$
        Update $UB = LB + \sum_s Z_{SP_s}$.
        Generate new columns ρ_s^j and add them to [RMP], $j \leftarrow j + 1$.
    end while
    Solve lower-level problems using information of $\mathbf{x}^*$ to get optimal production level $\hat{\mathbf{g}}_i$.
    $i \leftarrow i + 1$
end while

3.6 Numerical Experiments and Results

In this section, a series of numerical experiments are conducted and the results are analyzed. Our model and algorithms are tested on IEEE reliability testing systems [72]. Table 3.6 shows the instances including number of generators, number of wind generators and number of transmission lines of each testing systems.

Table 3.6: IEEE reliability testing systems

System	Generator	Wind Generator	Transmission lines
3bus	6	3	3
30bus	9	4	41
57bus	7	2	80
118bus	54	10	186

Our model and algorithm are tested in a four-stage ($T = 4$) planning horizon with each stage spans for 5 years, i.e. 43800 generation hours at each stage. Unless specificly stated, we assume the demand increases at the rate of 1% each year and the investment cost has an annual interest rate at 1%. The converging gaps for linear heuristic ϵ_1 and for decomposition ϵ_2 in Algorithm 1 are set to be 1×10^{-3} and 5×10^{-3}, respectively. The *endogenous* uncertainty level (see Section 3.3.1) is set at 20%. Each stage is divided into four demand blocks as mentioned in Section 3.3.1. The demand level at each demand block is set as 0.95, 0.85, 0.75, 0.65, respectively. The wind capacity factor levels at each demand block are set as 0.55, 0.45, 0.35, 0.25. The other settings and data, such as the specs of generators, the demand amount, the local wind intensity factor, are acquired from IEEE reliability testing systems or mentioned in the later sections.

The computational model is programmed in C++ by calling the commercial MILP solver of ILOG CPLEX 12.5. All experiments are implemented on a personal computer, which has quad Intel Core i7 processors with CPU at 3.40 GHz and a RAM space of 8GB.

3.6.1 Investment Analysis of IEEE 3 Bus System

Our model is analyzed using a simple 3-bus testing system acquired from [49], shown in Figure 3.6. This test network consists of three nodes and three transmission lines. At each node, there are a thermal generator already installed. The demand is also connected at each node. The entire network is divided into two wind zones that has different wind characters. Our model is applied to compute the optimal investment decisions on generator's type, size, and location.

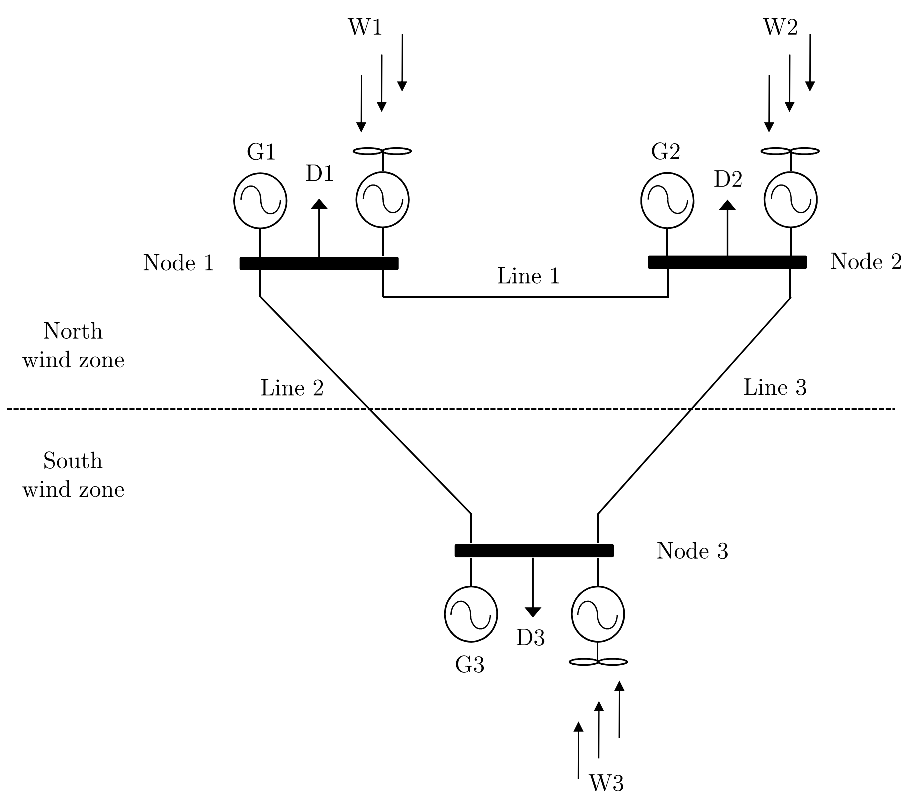

Figure 3.6: IEEE 3 bus testing system

The data of all generators, both existing or invested, are listed in Table 3.7. Units G1, G2 and G3 are existed thermal generator units at node 1, 2 and 3, respectively. The generator W1, W2, W3 are wind generators that have not been constructed yet. The wind generator has zero fuel cost. The investment budget is set at 40 Million $.

Table 3.7: Generator Data of IEEE 3 bus System

Unit	Type	Location	Current Capacity [MW]	Max Capacity [MW]	Fuel Cost [$/MWh]	O&M Cost [$/MWh]	Investment Cost [$/MW]
G1	Thermal	Node 1	150	200	31.67	26.24	150000
G2	Thermal	Node 2	150	200	64.16	8.34	120000
G3	Thermal	Node 3	100	150	39	13.29	184600
W1	Wind	Node 1	0	100	0	15.26	200000
W2	Wind	Node 2	0	100	0	15.26	200000
W3	Wind	Node 3	0	100	0	15.26	200000

The capacity factor represents the ratio of capacity that can be used for production. We assume all of the thermal generators has the same capacity factor of 0.85. On the other hand, the capacity factor of wind generator depends on wind conditions. The wind condition is both local and seasonal dependent [36]. The seasonal dependency is reflected by wind capacity factor levels for each demand block, mentioned in Section 3.3.1. The value of wind capacity factor levels are provided in Section 3.6. The local dependency is related to the geographic conditions of specific power network. In this 3 bus system, we assume that the wind speed in the north wind zone is lower than the wind speed in south wind zone. Therefore, the capacity factor of wind generator at node 3 is higher than those in node 1 and 2. The capacity factor data is shown in Table 3.8

Table 3.8: Capacity Factor of IEEE 3 bus System

Unit	Capacity Factor [p.u.]
G1, G2, G3	0.85
W1, W2	0.4*
W3	0.5*

$*$: The average value of capacity factors over four demand blocks

As discussed in Section 3.3.1, the uncertain demand follows normal distribution with the mean (D) and standard deviation (Std), provided in Table 3.9. We assume the demand increases at the rate of 2% each year.

Table 3.9: Demand of IEEE 3 bus System

Demand	D [MW]	Std [MW]
D1	120	20
D2	100	20
D3	100	20

Table 3.10: Transmission line data of IEEE 3 bus System

Line	From node	To node	S [p.u.]	f^{max} uncongested [MW]	f^{max} congested [MW]
1	1	2	5	100	30
2	1	3	5	100	30
3	2	3	5	100	30

The transmission line data are provided in Table 3.10. We test our model under two transmission conditions: uncongested and congested. The uncongested network's transmission line has enough capacity to transmit generated power. On the other hand, in the congested network, the transmission limit capacity is limited. The constraints of transmission limitation will be binding in the solution process.

The uncongested result is shown in Table 3.11. Note that Table 3.11 only illustrates the investment decision solutions for one of the uncertain scenario. The result shows that the investment involves on both thermal and wind generators. The wind generator is more preferable for investors due to its low cost advantage. We notice that the node 3 has the most investment due to the wind intensity in node 3 is larger than node 1 and 2. This result indicates that the investment decision provided by our model takes both sizing and siting into consideration. We also notice that the investment decision covers all available stages (our settings prohibits investment on Stage 4) to advocate the growing demand and to minimize the construction cost. This result shows that it is necessary to consider the multistage framework for long-term investment planning.

Table 3.11: Result for uncongested network of IEEE 3 bus system

Unit	Investment Decision [MW]				Expected Profit ($)
	Stage 1	Stage 2	Stage 3	Stage 4	
G1	11	0	0	0	2.95E+08
G2	0	0	26	0	
G3	0	0	0	0	
W1	0	1	0	0	
W2	0	0	0	0	
W3	73	27	0	0	

Table 3.12: Result for congested network of IEEE 3 bus system

Unit	Investment Decision [MW]				Expected Profit ($)
	Stage 1	Stage 2	Stage 3	Stage 4	
G1	0	0	5	0	2.79E+08
G2	0	0	44	0	
G3	0	0	0	0	
W1	0	0	0	0	
W2	6	0	0	0	
W3	72	10	0	0	

The investment decisions of congested network is recorded in Table 3.12. Comparing to results from uncongested network, the congested case has less wind investment in node 3, even though the wind power has advantage at node 3. This is because the transmission limits the power flow within the network, and become the bottleneck. The investment decision has adjust the investment decisions to make sure the investment decision is feasible. This compromise is also reflected on the decrease of total expected profit comparing to the uncongested case.

3.6.2 Computation time comparison

In Section 3.5.4, we employed the Dantzig-Wolfe decomposition algorithm to address the computational challenges in our multistage stochastic model.

We conduct computation on different size of electricity systems to compare the performance of proposed decomposition algorithm to the one from directly solving the problem (3.11). The comparison of computation time is shown in Table 3.13. The column “Bus” refers to the IEEE 3 bus, 30 bus, 57 bus and 118 bus testing systems, respectively. The column “Direct Solving” refers to the computation time used to by using commercial solver (CPLEX) to solve the problem directly. The column “Decomposition” refers to the computation time from using Dantzig-Wolfe decompostion algorithm. We assume the demand increases at the rate of 1% each year. The investment budget is set at 40 Million $ for the computations in Table 3.13.

Table 3.13: Computation time: Direct vs Decomposition

Bus	Computation time (sec)	
	Direct Solving	Decomposition
3	22.78	58.39
30	64558.26	58875.85
57	(4.34%)	959.62
118	(4.15%)	(2.19%)

Note: The percentage in parenthesis represents
the relative gap when the 80000s time limit is reached

From the result, we notice that for small systems, i.e. 3 bus, the problem is solved faster from direct solve approach. When it comes to larger systems, i.e. 57 bus, the solution gap of direct solve cannot converge with 80000 seconds. Both methods cannot solve the 118 bus system within the time limit, but the decomposition approach provides a better gap then direct solve. This results shows the computational difficulty can be well managed by the proposed solution approach.

3.6.3 Decision-Dependent Analysis

Similar to the study in Chapter 2, we compute the value of decision-dependent stochastic programming solution (VDDSS) to examine the effectiveness of the decision-dependent approach. Recall the discussion in Chapter 2, the VDDSS is calculated by first acquiring the optimal solution from a traditional stochastic model. Then, this solution is plugged into the decision-dependent formulation and the objective function value is then acquired. Finally, the VDDSS is calculated as the difference between the optimal objective value from decision-dependent approach and the objective value by using the traditional stochastic model solution in the decision-dependent model. The computation is conduct on 3 Bus testing system, where the investment budget is set at 10 Million $.

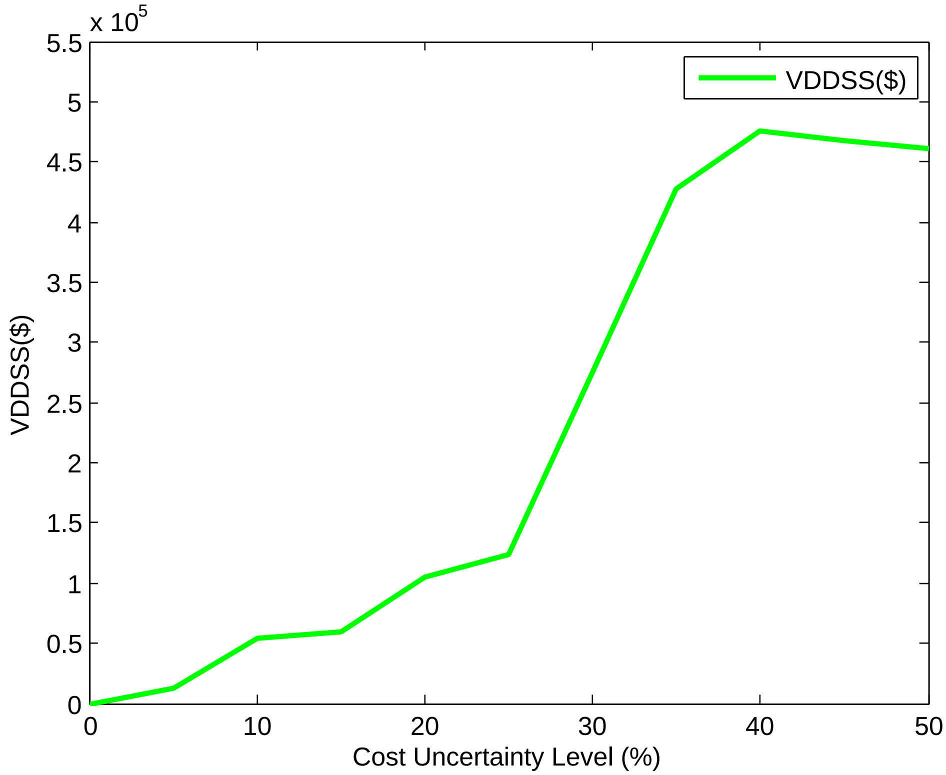

Figure 3.7: The VDDSS on 3 Bus System

Figure 3.7 shows that as cost uncertainty changes from 0% to 50%, the VDDSS increases dramatically. When the cost variation level is equal to zero, we observe that VDDSS is also equal to zero. This is because they both reduce to the same deterministic model. We observe that the VDDSS is greater than zero which indicates that the optimal solution from decision-dependent formulation provides a larger profit than the one from the traditional stochastic programming approach. From the above observation, we can conclude that it is important to take into account the decision-dependent approach for long-term investment planning problems.

3.6.4 Computation Results

Table 3.14: Computation Results for IEEE 3, 30, 57 and 118-bus systems

Bus	Budget (Million $)	Time (sec)/Gap	Optimal ($)
3	10	333.71	9.64E+07
	20	39.35	2.75E+08
	30	156.45	3.72E+08
	40	78.54	4.36E+08
	50	404.45	5.27E+08
30	10	2793.98	6.29E+07
	20	4117.52	1.43E+08
	30	34953.79	1.98E+08
	40	58875.85	2.20E+08
	50	(1.56%)	2.18E+08
57	20	802.06	9.66E+07
	30	1035.38	1.48E+08
	40	959.62	1.56E+08
	50	1910.91	1.57E+08
118	10	1385.84	4.94E+07
	20	3572.46	9.87E+07
	30	44034.73	1.40E+08
	40	(2.19%)	1.60E+08
	50	(11.88%)	1.78E+08

Note: The percentage in parenthesis represents
the relative gap when the 80000s time limit is reached

Table 3.14 exhibits the computational results from solving all the tested systems. The column "Budget" refers to the total investment budget of all stages. The column "Time/Gap" gives the total computation time for solving each case or the gap between upper and lower bounds when the time limit is reached. The column "Optimal" refers to the optimal value of the expected total profit. We observe that most of the test problems can be solve within time limit (80000s). As the size of electricity system increases, the computation takes longer time to solve. This is because the problems with largest electricity system have more constraints. We also notice that as the investment budget increases, the computation also takes longer time. This is because the problems with larger investment budget has larger feasible region, thus the binary variables' branch-and-bound tree has more nodes that usually leads to longer solution process.

CHAPTER 4: MULTISTAGE ADAPTIVE ROBUST OPTIMIZATION FOR POWER GENERATION EXPANSION PLANNING

4.1 Introduction

The model electricity system involves in a lot of uncertain factors such as wind speed, water inflows, the users' demand and so on. Dealing with uncertainty has always been a challenging task for electricity power investors.

Recall the discussion in Chapter 1, stochastic programming has been widely applied to manage the uncertainty for long-term generation expansion planning problems. However, stochastic programming also faces great challenges to identify appropriate probability distributions. Moreover, the size of the scenario tree could be very large, especially for a high-dimensional uncertain process. The large size scenario trees often lead to great computational complexity [77].

Robust optimization is another widely applied approach for optimization problems under uncertainty. Instead of solving for the expected optimal of uncertain scenarios, robust optimization acquires the optimal under the worst case scenario [78]. The uncertainty data in the robust model are realized as given *uncertain sets*. When the uncertain data "drift" around their nominal values, the optimal solutions can be heavily affected and even may cause infeasibility. Robust optimization deals with this type of uncertain and seeks for an optimal solution that remains feasible for all realization of uncertain data [5].

However, robust optimization sometimes encounters over-conservative issue. Due to the fact that robust optimization requires all the decisions be made before the actual realization of uncertain data. However, there are plenty of real-world cases that only part of decisions need to be made in advance. Hence, the robust optimization approach is not an accurate presentation of real-world cases. For example, in two or multistage cases, some type of variables are categorized as *wait-and-see* decision variables that can be determined until the uncertainty is unfolded in the future stage. But the robust optimization enforces all the *wait-and-see* variables to be determined at the very beginning, which makes the solution over-conservative. Under this context, Ben-Tal et al. [5] introduced the concept of adaptive robust optimization approach, as an extension of robust optimization methodology, to address the over-conservative issue. For LP structure problems, the linear robust optimization approach is also denoted as robust counterpart (RC) and the linear adaptive robust optimization approach is denoted as adjustable robust counterpart (ARC) [5]. In the adaptive robust optimization approach, it waits until the uncertain data is unfolded, then determines the optimal value of the *wait-and-see* variables, this process is exactly like their names "wait-and-see".

The adaptive robust optimization approach has more flexibility than robust optimization approach, but this flexibility also brings in great computational challenges that making the optimization problem computational intractable in most cases. This computational challenge is addressed by introducing *affine* policy [5] for the problem with linear structure. The *affine* policy assumes that the "wait-and-see" variables can only be affinely adjustable to the uncertain data.

The rest of this chapter is arranged as follows. Section 4.2 reviews the existing methodology and applications, and point out the research gaps that will be addressed in our research. In Section 4.3, we study the long-term power generation investment expansion planning problem by presenting a deterministic multistage optimization model. The limitations of deterministic model is discussed. Section 4.4 propose a multistage robust long-term investment planning model. Section 4.5 presents solution methods by adopting the simplied affine policy for our model. Section 4.6 presents numerical computational study of the performance of the proposed approach.

4.2 Literature Review

Led by the work in [79–86] , robust optimization has been recently gained substantial popularity as a modeling framework for optimization under uncertainty. It provides several features that are particular appealing to the applications of optimization. First, the robust framework only requires moderate information about the uncertainty of the data. This feather enable the robust optimization to acquire optimal especially when accurate probability distributions are not able to obtain [86]. Second, by the feature of robustness of the optimization model, the optimal solution immunizes against all realizations of the uncertain data within a deterministic uncertainty set [84].

A popular extension of robust optimization, i.e. adaptive robust counterpart (ARC), is introduced by Ben-Tal et. al. [5]. It separates the *adjustable* variables and *non-adjustable* variables during the optimization process and therefore release the flexibility of the *wait-and-see* variables. This feature is extremely important for power system. Because many of optimization problems in power system contain both *here-and-now* variables (e.g. commitment decisions) and *wait-and-see* variables (e.g. generation production). Several cutting edge studies in power system have adopted adaptive robust optimization approach. Jiang et al [87] present a two-stage adaptive robust model to deal with a formulation including pumped storage hydro with wind power uncertainty. Zhao & Zeng [88] tackle the unit commitment problem to obtain the day-ahead generator schedules with considering wind uncertainty. Bertsimas et. al. [84] tackle the security constrained unit commitment (SCUC) problem by a two-stage adaptive robust optimization model, with commitment decisions in the first stage and dispatch decisions in the second stage. Hybrid models and alternative objectives are explored in [89] to ensure the robustness of the unit commitment decision considering the inherent uncertainty in wind generation. The work in [86] brought us two-stage adaptive robust optimization model with a dynamic uncertainty set that explicitly model temporal and spatial correlations in variable sources.

The adaptive robust optimization is computational challenging. Unlike the stochastic programming that has finite numbers of scenarios, the uncertainty of robust optimization is based on the continuous uncertain set, and therefore they have infinite numbers of possibilities. The computational challenge of adaptive robust optimization has been addressed with the approximations of the decision rules. The *affine* policy is introduced in [5] to restrict the *wait-and-see* variables to be only be affinely adjustable to the uncertain data. Bertsimas et al. [85] studies the *affine* policy which uses the connections between the geometrical properties of the feasible sets and the objective functions. This approach theoretically proves that the multistage adaptive robust decision problems is computational tractable with *affine* policy.

4.2.1 Research Gap

If we try to summarize the above works, we can draw the following observations of research gaps: 1) The applications of robust optimization in multistage framework is very rarely. To the best of our knowledge, there is no study applying multistage adaptive robust optimization framework to the long term generation investment planning problem. 2) To the best of our knowledge, in the previous studies of generation investment planning problems, the investment decisions are considered as *non-adjustable* which must be determined before the uncertainty unfolds, which is too conservative.

To address these research gaps, we propose a multi-stage adaptive robust optimization model for long term generation investment planning problem. In our model, the investment decision variables are considered as *adjustable* along with the planning stages.

4.3 Deterministic Model for Multi-stage Power Generation Expansion Planning Problem

In this section, we discuss the deterministic multi-stage generation expansion planning model aiming at maximize the total profit. The decision variables are expansion decisions and generation decisions. Our model assumes the planning and generator are conducted within a DC power network. The electricity network consists two types of generators units: thermal and wind, which is typical for the electricity network in the middle west. We also assume that the storage units are not considered in the network. This is because the energy storage units are mainly applied to deal with energy dispatch problems in the short-term market such as day-ahead unit commitment [31]. Our long-term planning horizon averages out the effect of storage units in the short-term.

The sets and indices are listed in Table 4.1. The parameters are listed in Table 4.2.The variables are listed in Table 4.3

Table 4.1: Indices and Sets for Deterministic Model

$\mathcal{N}_g$	Set of generators.
$\mathcal{N}_l$	Set of transmission lines.
$\mathcal{N}_d$	Set of demand nodes.
$\mathcal{T}$	Set of time periods (or stages).
$t = 1, \ldots, T$	Index of time periods (or stages).
i	Index of generator, $i \in \mathcal{N}_g$.
j	Index of demand node, $j \in \mathcal{N}_d$.
l	Index of transmission line, $l \in \mathcal{N}_l$.

$$\max \sum_{i \in \mathcal{N}_g} \left[\sum_{t \in \mathcal{T}} [(p - c_i) g_i^t - m_i x_i^t] - B_i \cdot (x_i^T - x_i^{ini}) \right] \tag{4.1a}$$

$s.t.$ Capacity expansion relation

$$x_i^t - x_i^{t-1} \geq 0, \quad \forall i \in \mathcal{N}_g,\ t \in \mathcal{T} \tag{4.1b}$$

Table 4.2: Parameters for Deterministic Model

p	Unit selling price.
B_i	Unit expansion cost for generator i.
c_i	Unit generation cost for generator i.
m_i	Unit operation and maintenance (O&M) cost for generation i.
x_i^{ini}	Existing capacity for generator i.
$x_i^{\max}$	Maximum allowed capacity for generator i.
β_i	Capacity factor for generator i.
$f_l^{\max}$	The capacity for transmission line l.
H	The number of hours in one time period/stage.
d_j^t	The demand level at node j at time period/stage t.
$\boldsymbol{S}_l^p$	The Generation shifting factor matrix.
$\boldsymbol{S}_l^d$	The demand shifting factor matrix.

Table 4.3: Variables for Deterministic Model

x_i^t	Capacity for generator i at stage t.
g_i^t	Power production for generator i at stage t.

Capacity limitation

$$x_i^{ini} \leq x_i^t \leq x_i^{\max}, \quad \forall i \in \mathcal{N}_g,\ t \in \mathcal{T} \tag{4.1c}$$

Generation limitation constraint

$$0 \leq g_i^t \leq H\beta_i x_i^t, \quad \forall i \in \mathcal{N}_g,\ t \in \mathcal{T} \tag{4.1d}$$

Energy balance constraint

$$\sum_{i \in \mathcal{N}_g} g_i^t = H \sum_{j \in \mathcal{N}_d} d_j^t, \quad \forall t \in \mathcal{T} \tag{4.1e}$$

Transmission line limits

$$-Hf_l^{max} \leq (\mathbf{S}^g \mathbf{g}^t - H\mathbf{S}^d \mathbf{d}^t)_l \leq Hf_l^{max}, \quad \forall l \in \mathcal{N}_l,\ t \in \mathcal{T} \tag{4.1f}$$

Nonegativity restrictions

$$g_i^t, x_i^t, \quad \forall i \in \mathcal{N}_g,\ t \in \mathcal{T} \geq 0 \tag{4.1g}$$

The objective is to maximize the expected profit, which is calculated as the difference between the total generation income and total cost. The decision variables are expanded capacity x_i^t and generated electricity g_i^t of generator i at stage t. In the objective function (4.1a), the term $(p - c_i)g_i^t$ represents the generation income for generator i at stage t, where p is the selling price, c_i is the generation cost and g_i^t represents the generation production. The term $m_i x_i^t$ corresponds to operation and maintenance (O&M) cost, and $B_i(x_i^T - x_i^{ini})$ represents the expansion investment cost, where B_i is the unit expansion cost. x_i^{ini} and x_i^T represent the capacity of generator i before any investment and the capacity after all the investment at the last stage T, respectively. This model subjects to a series of constraints representing technical conditions of the real-world DC network. Constraints (4.1b) states that the generator's capacity of stage t should be not less than the capacity in its previous stage. Constraints (4.1c) bounds the generators capacity within its initial capacity and its maximum allowed capacity. The fact that generated electricity should not exceed the available capacity, is stated in constraint (4.1d). The energy balance constraints (4.1e) ensure the total demand is satisfied. The power transmission is limited by transmission line limitations in (4.1f). Finally, constraints (4.1g) ensure the generation and capacity are nonnegative.

The limitation of the deterministic model (4.1) is obvious. The optimization solution is based on the perfect knowledge of future market demand. As we have discussed in Chapter 1, the electricity system has the nature of uncertainty, thus it is difficult to make precise prediction of future demand. If the demand in the future stages is different than its nominal value, the optimal solution provided by the deterministic model will become infeasible which may causes catastrophic disaster for the power system. This limitation of the deterministic model motivate us to consider multistage robust models.

4.4 Multi-stage Adaptive Robust Generation Expansion (MARGE) Model

In this section, we propose the multistage adaptive robust generation expansion (**MARGE**) model to deal with the demand uncertainty in long-term generation expansion planning problem. We first set the uncertainty in this model, then present the formulation.

4.4.1 Uncertainty Setting

In this chapter, we assume the uncertain demand obeys following box uncertainty,

$$\mathcal{D}^t = \left\{ \mathbf{d}^t = (d_1^t, \ldots, d_{N_d}^t : d_j^t \in [\bar{d}_j^t - \hat{d}_j^t, \bar{d}_j^t + \hat{d}_j^t], \ \forall j \in \mathcal{N}_d) \right\}. \ \forall t \in \mathcal{T} \tag{4.2}$$

where $\mathbf{d}^t$ is the vector of demand at all nodes and at stage t, $\mathcal{D}^t$ represents the uncertain set of demand. Notice that d_j^t lies in an interval centered around the nominal value $\bar{d}_j^t$ within a deviation denoted by $\hat{d}_j^t$. We define $\mathcal{D} = \prod_{t \in \mathcal{T}} \mathcal{D}^t$ as the uncertainty set for the demand over entire planning horizon.

4.4.2 Model Formulation

Next, we are going to formulate the adaptive robust optimization model base on the deterministic model in (4.1). In the deterministic model (4.1), there are two decision variables: capacity x_i^t and power production g_i^t. There are plenty of studies [77, 84, 86] showing that the power production belongs to *adjustable* variables. This is because the generation is almost instantaneous adjustable according to the demand change. On the other hand, the capacity (investment) variables are treated as *non-adjustable* in most two-stage adaptive robust optimization models. However, in the multistage framework, the investment decisions for generation units can adjust to some extent before the construction start. To faithfully model this process, the capacity (investment) variables are set to be *adjustable*.

In the adaptive robust framework, the power production $\mathbf{g}^t$ and the capacity $\mathbf{x}^t$ at time t should depend on the history of market demand $\mathbf{d}^{[t]} \triangleq (\mathbf{d}^1, \ldots, \mathbf{d}^t)$. We formulate the following multistage adaptive robust generation expansion planning (MARGE) model.

(MARGE)

$$\max_{\boldsymbol{x},\boldsymbol{g}} \min_{\mathbf{d}\in\mathcal{D}} Z^* = \sum_{i\in\mathcal{N}_g} \left\{ \sum_{t\in\mathcal{T}} [-m_i x_i^t(\mathbf{d}^{[\mathbf{t}]}) + (p - c_i) g_i^t(\mathbf{d}^{[\mathbf{t}]})] - B_i(x_i^T(\mathbf{d}^{[\mathbf{t}]}) - x_i^0) \right\} \tag{4.3a}$$

$$s.t.\ x_i^0 \le x_i^t(\mathbf{d}^{[t]}) \le x_i^{\max}, \quad \forall d \in \mathcal{D},\ i \in \mathcal{N}_g,\ t \in \mathcal{T} \tag{4.3b}$$

$$x_i^{t-1}(\mathbf{d}^{[\mathbf{t-1}]}) - x_i^t(\mathbf{d}^{[\mathbf{t}]}) \le 0, \quad \forall d \in \mathcal{D},\ i \in \mathcal{N}_g,\ t \in \mathrm{T} \tag{4.3c}$$

$$g_i^t(\mathbf{d}^{[\mathbf{t}]}) \le H\beta_i x_i^t(\mathbf{d}^{[\mathbf{t}]}), \quad \forall d \in \mathcal{D},\ i \in \mathcal{N}_g,\ \forall t \in \mathrm{T} \tag{4.3d}$$

$$\sum_{i\in\mathcal{N}_g} g_i^t(\mathbf{d}^{[\mathbf{t}]}) = H \sum_{j\in\mathcal{N}_d} d_j^t, \quad \forall d \in \mathcal{D},\ i \in \mathcal{N},\ t \in \mathrm{T} \tag{4.3e}$$

$$-Hf_l^{max} \le (\mathbf{S}^g \mathbf{g}^t(\mathbf{d}^{[\mathbf{t}]}) - H\mathbf{S}^d \mathbf{d}^t)_l \le Hf_l^{max}, \quad i \in \mathcal{N}_l,\ \forall t \in \mathcal{T} \tag{4.3f}$$

$$g_i^t(\mathbf{d}^{[\mathbf{t}]}), x_i^t(\mathbf{d}^{[\mathbf{t}]}) \ge 0, \quad \forall d \in \mathcal{D},\ i \in \mathcal{N}_g,\ \forall t \in \mathrm{T} \tag{4.3g}$$

In this formulation, the demand $\mathbf{d}$ is no longer a fixed parameter. Instead, it is changeable in the uncertain set $\mathcal{D}$. The "max-min" combination in the objective function (4.3a) is called *robust counterpart*. The minimization term $\min_{\mathbf{d}\in\mathcal{D}}$ seeks for the worst-case realization of uncertain $\mathbf{d}$. Thus, this process guarantees an optimal objective function value not worse than Z^*. The optimal solution x^*, g^* should satisfy the constraints for *all* possible realizations of $d \in \mathcal{D}$, known as the *robust*.

At this point, the **MARGE** model is computational intractable because the decision rules for the *adjustable* variables $\mathbf{g}^t(\cdot)$ and $\mathbf{x}^t(\cdot)$ are unknown. In the following, we propose approximated decision rules and tractable solution methods for **MARGE** model.

4.5 Solution Method

In this section, we first introduce the affine policy to address the computational challenge of **MARGE** model. Then, the multistage affinely adaptive robust model for generation expansion (MAARGE) is presented. The solution consideration is then discussed.

4.5.1 Affine Policy

Since our **MARGE** model is formulated based on linear deterministic model (4.1), it is natural to assume that the decision rules for *adjustable* variables $\mathbf{g}^t(\cdot)$ and $\mathbf{x}^t(\cdot)$ should also be in linear (affine) forms. The *affine* policy introduced by Ben-Tal [5] has proven to be computational tractable for adaptive robust models.

In this research, we take advantage of the affine function in [77] to model the decision rules of variables $\mathbf{g}^t(\cdot)$ and $\mathbf{x}^t(\cdot)$, as follows.

$$g_i^t(\mathbf{d}^{[t]}) = w_i^t + \sum_{j \in \mathcal{N}_d} \sum_{\tau \in [1:t]} W_{itj\tau} d_j^\tau, \quad \forall i \in \mathcal{N}_g, \ \forall t \in \mathrm{T} \tag{4.4a}$$

$$x_i^t(\mathbf{d}^{[t]}) = v_i^t + \sum_{j \in \mathcal{N}_d} \sum_{\tau \in [1:t]} V_{itj\tau} d_j^\tau, \quad \forall i \in \mathcal{N}_g, \ \forall t \in \mathcal{T} \tag{4.4b}$$

where τ represents the history information from stage 1 to t, $(w_i^t, W_{itj\tau})$ and $(v_i^t, V_{itj\tau})$ are the coefficients of the affine policy.

4.5.2 Simplified Affine Policy

The study in [77] proposed simplified version of affine policy. The degrees of freedom of the coefficients are reduced. According to the numerical study in [77], it turns out the simplified affine policy provides "surprisingly well" as approximate solutions to the full adaptive problem. Therefore, we adopt the simplified affine policy as follows,

$$g_i^t(\mathbf{d}^{[t]}) = w_i^t + W_{it} \sum_{j \in \mathcal{N}_d} d_j^t, \quad \forall i \in \mathcal{N}_g,\ \forall t \in \mathrm{T} \tag{4.5a}$$

$$x_i^t(\mathbf{d}^{[t]}) = v_i^t + V_{it} \sum_{j \in \mathcal{N}_d} d_j^t, \quad \forall i \in \mathcal{N}_g,\ \forall t \in \mathcal{T} \tag{4.5b}$$

4.5.3 Multistage Robust GE Model with Affine Policy

We replace the adjustable variables $\mathbf{g}^t(\cdot)$ and $\mathbf{x}^t(\cdot)$ with the simplified affine policies (4.5). The multistage affinely adaptive generation expansion planning (MAARGE) model is formulated as follows,

(**MAARGE**)

$$\max_{\boldsymbol{Z},\boldsymbol{w},\boldsymbol{W},\boldsymbol{v},\boldsymbol{V}} Z \tag{4.6a}$$

$$\begin{aligned} s.t.\ & \sum_{i \in \mathcal{N}_g} \Bigg\{ \sum_{t \in \mathcal{T}} \Bigg[-m_i(v_i^t + V_{it} \sum_{j \in \mathcal{N}_d} d_j^t) + (p - c_i)(w_i^t + W_{it} \sum_{j \in \mathcal{N}_d} d_j^t) \Bigg] \\ & -B_i(v_i^T + V_{iT} \sum_{j \in \mathcal{N}_d} d_j^T + x_i^0) \Bigg\} \geq Z,\ \forall d \in \mathcal{D} \end{aligned} \tag{4.6b}$$

$$\sum_{i \in \mathcal{N}_g} B_i^t \cdot \left(v_i^T + V_{iT} \sum_{j \in \mathcal{N}_d} d_j^T - x_i^0 \right) \leq B^{max},\ \forall d \in \mathcal{D} \tag{4.6c}$$

$$x_i^0 \leq \left(v_i^t + V_{it} \sum_{j \in \mathcal{N}_d} d_j^t \right) \leq x_i^{\max}, \quad \forall d \in \mathcal{D},\ i \in \mathcal{N}_g,\ t \in \mathcal{T} \tag{4.6d}$$

$$(v_i^{t-1} - v_i^t) + (V_{i,t-1} \sum_{j \in \mathcal{N}_d} d_j^{t-1} - V_{it} \sum_{j \in \mathcal{N}_d} d_j^t) \leq 0, \quad \forall d \in \mathcal{D},\ i \in \mathcal{N}_g,\ \forall t \in \mathcal{T} \tag{4.6e}$$

$$w_i^t + W_{it} \sum_{j \in \mathcal{N}_d} d_j^t \leq H\beta_i \left(v_i^t + V_{it} \sum_{j \in \mathcal{N}_d} d_j^t \right), \quad \forall \mathbf{d} \in \mathcal{D}, i \in \mathcal{N}_g, t \in \mathcal{T} \tag{4.6f}$$

$$\sum_{i \in \mathcal{N}_g} (w_i^t + W_{it} \sum_{j \in \mathcal{N}_d} d_j^t) = H \sum_{j \in \mathcal{N}_d} d_j^t, \quad \forall d \in \mathcal{D},\ t \in \mathcal{T} \tag{4.6g}$$

$$-Hf_l^{max} \leq \sum_{i \in \mathcal{N}_g} S_{li}^g (w_i^t + W_{it} \sum_{j \in \mathcal{N}_d} d_j^t) + H \sum_{j \in \mathcal{N}_d} S_{lj}^d d_j^t \leq Hf_l^{max},\ \forall d \in \mathcal{D},\ i \in \mathcal{N}_l,\ t \in \mathcal{T} \tag{4.6h}$$

$$v_i^t + V_{it} \sum_{j \in \mathcal{N}_d} d_j^t \geq 0, \quad \forall d \in \mathcal{D},\ i \in \mathcal{N}_g,\ \forall t \in \mathcal{T} \tag{4.6i}$$

$$w_i^t + W_{it} \sum_{j \in \mathcal{N}_d} d_j^t \geq 0, \quad \forall d \in \mathcal{D},\ i \in \mathcal{N}_g,\ \forall t \in \mathcal{T} \tag{4.6j}$$

In this model, we create a new variable Z to get rid of the “max-min” formulation. The constraints (4.6b) denote the worst-case total profit.

4.5.4 Solution Approach

The robust constraints in (4.6) have following structure:

$$c(\mathbf{V}, \mathbf{W})\mathbf{d} \leq h(\mathbf{v}, \mathbf{w}, Z),\ \forall \mathbf{d} \in \mathcal{D} \tag{4.7}$$

Both $\mathbf{V}, \mathbf{W}$ and $\mathbf{d}$ are actually treated as variables in **MAARGE** model. At first glance, the constraints (4.7) contains bilinear terms, which make our model to be nonlinear and nonconvex. However, after closer examination, we notice that even though $\mathbf{d}$ is a variable value, its uncertainty should always be realized to provide the "worst" objective value. In another word, the uncertain $\mathbf{d}$ is *worst case oriented*. Therefore, the uncertain demand $\mathbf{d}$ cannot be treated as a regular free variable, but should be regarded as a "solution" $\mathbf{d}^*$ when we seek for the optimal of $\mathbf{y} = \{\mathbf{v}, \mathbf{w}, \mathbf{V}, \mathbf{V}, Z\}$.

We notice that the left-hand side of (4.7) is a linear function in $\mathbf{d}$ and the uncertainty set $\mathcal{D}$ is a polytope. Hence, the "solution" $\mathbf{d}^*$ should always be at the extreme points of the set $\mathcal{D}$. Therefore, the robust constraint (4.7) is equivalent to an enumeration of all the extreme points of $\mathcal{D}$. The robust constraint (4.7) is equivalent to:

$$\mathbf{c}^\top \mathbf{d} \leq \mathbf{h},\ \forall \mathbf{d} \in ext(\mathcal{D}), \tag{4.8}$$

where $ext(\mathcal{D})$ represents all the extreme points of $\mathcal{D}$ [4]. This applies to every inequality constraints in **MAARGE**.

The energy balance constraint (4.6g), however, is an equality constraint that cannot apply for the relation in (4.8). Instead, the study in [77] discover an unique feather for this constraint. First, rewrite constraint (4.6g) as follows,

$$\sum_{i \in \mathcal{N}_g} w_i^t + \left(\sum_{i \in \mathcal{N}_g} W_{it} - H \right) \sum_{j \in \mathcal{N}_d} d_j^t = 0. \tag{4.9}$$

The equality should always hold for all $d \in \mathcal{D}$. Therefore, the value of w_i^t and W_{it} are enforced to be:

$$\sum_{i \in \mathcal{N}_g} w_i^t = 0 \tag{4.10a}$$

$$\sum_{i \in \mathcal{N}_g} W_{it} = H \tag{4.10b}$$

Thus, all of the uncertain d in (4.6) has been addressed. The model can be solved as a linear programming (LP).

4.6 Numerical Experiments and Results

In this section, a series of numerical experiments are conducted and the results are analyzed. We first use an illustrative example on a 2 bus testing system to compare the adaptive robust optimization approach against robust optimization and stochastic programming. Our model and algorithms are then tested on IEEE reliability testing systems [72]. Table 4.4 shows the instances including number of generators, number of wind generators and number of transmission lines of each testing systems.

Table 4.4: IEEE reliability testing systems

System	Demand node	Generator	Transmission lines
2 bus	2	1	1
4 bus	4	2	4
30bus	30	9	41
118bus	118	54	186

Our model and algorithm are tested in a four-stage ($T = 4$) planning horizon with each stage represents one year, i.e. 8760 generation hours at each stage. Unless specifically stated, the capacity factor for thermal and wind generators is set to be 0.85 and 0.35 respectively. The other settings and data, such as the specs of generators, the demand nominal value and deviation, are acquired from IEEE reliability testing systems or mentioned in the later sections.

The computational model is programmed in C++ by calling the commercial MILP solver of ILOG CPLEX 12.5. All experiments are implemented on a personal computer, which has quad Intel Core i7 processors with CPU at 3.40 GHz and a RAM space of 8GB.

4.6.1 Illustrative Example of 2 Bus System

We use an illustrative example to compare the results between 3 most popular optimization approaches for uncertainty: Stochastic optimizations (SO), Adaptive robust optimization/counterpart (ARC), and Robust optimization/counterpart (RC).

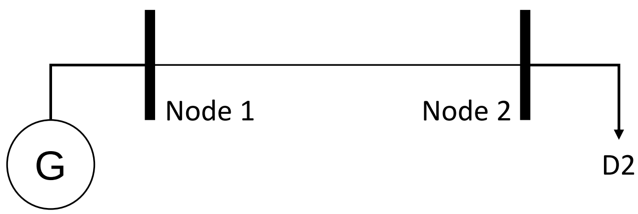

Figure 4.1: Simple two-bus system

Table 4.5: Data for 2 Bus testing system

Price	Cost			Generator Capacity		Transmission	Demand	
	Expansion	Generation	Maintenance	Initial	Max	Capacity	$\bar{d}$	$\hat{d}$
100	1.50E+07	31.67	26.24	150	400	600	200	40

Note: The power data are in unit of [MW], the price/cost data are in unit of \$/Mw

Figure 4.1, shows the network of the 2 bus system. The generator is connected to node 1 and the demand is connected to node 2. The data are shown in Table 4.5. For the sake of simplicity, we assume the capacity factor to be 1 in this example. From the data, we know that the nominal value for the demand is 200 MW, and its deviation is 40 MW. Thus, the uncertain range for the demand is $\mathcal{D}^t = [160, 240]$ for all the stages t.

The objective of stochastic programming model is set to seek for the maximum expected profit of the realizations of of uncertain demands. The model consists of a binary scenario tree with two child nodes connecting to each ancestor node. The realizations of uncertain demand at each node are set to be $\mathbf{d} \in ext(\mathcal{D})$. The probability of each scenario is set to be uniformly distributed. The RC model is actually a partial adaptive robust model. The decision variable of capacity are set to be *non-adjustable* and the generation decision remains to be *adjustable*.

The optimization results for SO, ARC and RC models are shown in Table 4.6, respectively. Their optimal objective values, i.e. the max profit, are -1000, -1169, and -1188 Million $, respectively.

Table 4.6: Optimization Result for SO, ARC and RC

Stochastic Results:

	Stage 1	Stage 2	Stage 3	Stage 4
Demand*	200	200	200	200
Generation*	200	200	200	200
Capacity*	200	220	230	235
Investment*	50	20	10	5

$*$: The results are expected value of stochastic scenarios.

ARC Results:

	Stage 1	Stage 2	Stage 3	Stage 4
Demand	160	160	160	160
Generation	160	160	160	160
Capacity	160	240	240	240
Investment	10	80	0	0

RC Results:

	Stage 1	Stage 2	Stage 3	Stage 4
Demand	160	160	160	160
Generation	160	160	160	160
Capacity	240	240	240	240
Investment	90	0	0	0

Note: The unit of all the data is [MW]

The maximum profit acquired from SO is the largest between three approaches. This is because the perspective of stochastic programming is to seek for the maximum expect value, whereas the robust approaches seek for the maximum profit under the worst-case scenario. The optimal value for ARC is larger than the optimal for RC. This observation shows the ARC is less conservative than RC. The investment decisions of ARC has less “waste” than RC because of the investment decision is *adjustable* in ARC. The *adjustable* investment plan is able to adjust its value for different realization of uncertain demands while still remaining the feasibility.

To test how the solutions from three approaches handling the uncertainty, we randomly assume the actual realization for uncertain demand is $\mathbf{d}^* = \{200, 240, 160, 160\}$, [MW]. We plug this $\mathbf{d}^*$ into the solutions from three different approaches to check if their solution remains feasible and has the largest profit.

Figure 4.2 shows the demand $\mathbf{d}^*$ comparing to the invested capacity. Because the stochastic programming only considers a finite number of possible uncertain realizations, it cannot cover all the possible realization of uncertain data. The solution from one scenario that has the closest value to $\mathbf{d}^*$ is shown in the first plot in Figure 4.2. We notice the demand exceeds the generators capacity of the first stage, therefore the capacity expansion plan from stochastic approach is infeasible. The second and the third plot in Figure 4.2 shows that both ARC's investment plan and RC's investment plan are feasible for the give $\mathbf{d}^*$. We notice that the RC approach made the largest investment at Stage 1 regardless of the demand level. This is because the capacity needs to prepare for the worst case. Since the capacity is treated as *non-adjustable* in RC, all the future capacity has to be determined before any realization of uncertainty. On the other hand, the capacity is set to be *adjustable* in ARC, thus the ARC only invested the needed amount. The profit from ARC's and RC's solution are -\$1107 and -\$1116, respectively. It also indicates the ARC approach is able reduce the investment and maintenance cost and therefore achieve a better optimal objective value, compared to the RC approach.

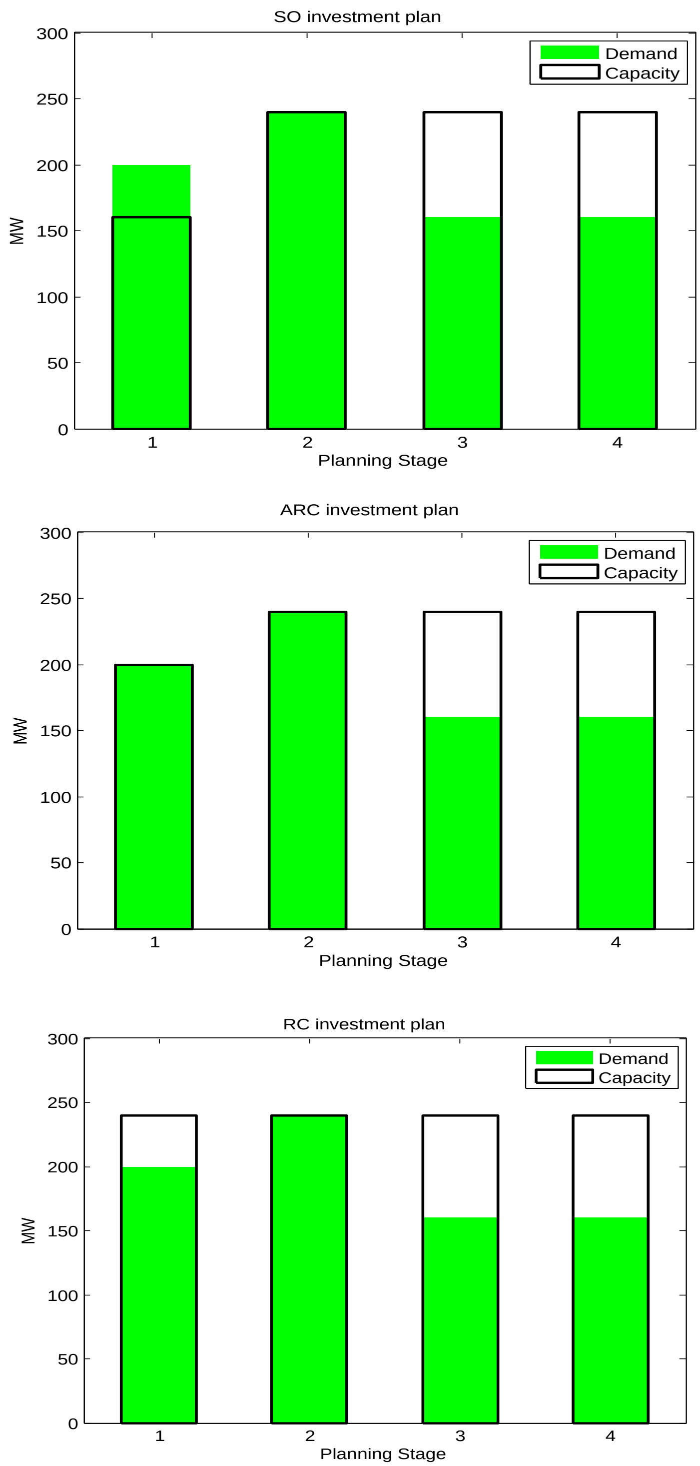

Figure 4.2: The demand vs capacity of the SO, ARC and RC approachs

4.6.2 *Computation Result for Optimal Objective Value*

Table 4.7: Optimal objective value of SO, ARC and RC

System	Uncertain Level	Stoch	ARC [million $]	RC
2 bus	0%	-455	-455	-455
	10%	-727	-812	-821
	20%	-1000	-1169	-1188
	30%	-1283	-1529	-1554
4 bus	0%	306	306	306
	10%	235	161	156
	20%	163	16	5
	30%	90	-132	-145
30 bus	0%	252	252	252
	10%	211	177	176
	20%	153	85	83
118 bus	0%	9100	9100	9100
	10%	8833	8177	8166
	20%	8024	6524	6477
	30%	7152	4769	4682

Table 4.7 exhibits the optimal objective values of SO, ARC and RC models by solving all the tested systems under different uncertainty levels. The uncertain level refers to a value δ being used configure the uncertain set $\mathcal{D}$. In this experiment, the demand's deviation level $\hat{d}_j^t$ is defined as a certain percentage of its nominal value: $\hat{d}_j^t = \delta \cdot \bar{d}_j^t$, e.g. when uncertain level is $\delta = 10\%$, then $\hat{d}_j^t = 10\% \cdot \bar{d}_j^t$.

When $\delta = 0\%$ the optimal values of three approaches are the same for all test cases. This is because both SO, ARC and RC reduce to the same deterministic model. We also observe that the difference of the optimal value becomes greater when the uncertain level is larger. This reflects the difference between the three approaches becomes greater when the uncertainty is larger.

CHAPTER 5: CONCLUSIONS

This dissertation studies serval optimization problems for long-term electricity power system's investment expansion planning. The long-term planning horizon and high penetration of wind energy brings in the uncertainty to the electrical power system. This dissertation uses multiple optimization methodologies to solve the investment planning problems under uncertainty.

In Chapter 2, we propose a decision-dependent stochastic programming model for long-term power generation expansion planning, where probabilities of price outcomes are variables dependent on investment decisions. We develop an optimization strategy to maximize the total profit. The decision-dependent probability distribution, which is one of the key features of our optimization model, is specified by the Return on Investment model and the Luce's probability model. We also link the demand to market price via the elasticity relationship. To solve this nonlinear stochastic program, a quasi-exact solution approach is then adopted to reformulate the multistage, stochastic, nonlinear model to a MILP model, which is solved by CPLEX. Our model and algorithm are then tested on four-stage case studies, which are based on a 20-year horizon. From the analysis of numerical results, we discover that generation expansion investment plays an important role in determining the probability distribution. Therefore, the proposed decision-dependent stochastic programming model, which adopts the decision-dependent probabilities, can provide effective optimization information on investment for long-term generation expansion planning.

In Chapter 3, a bi-level multistage decision-dependent stochastic programming model is proposed to solve for the long-term power generation investment expansion planing problem considering the market framework. This model seeks for the optimal sizing and siting for both thermal and wind power units to be built to maximizing the expected profit for a profit-oriented power investor. The proposed formulation is based on the bilevel framework that includes an upper-level stochastic expansion planning problem and a collection of lower-level problems that solves for optimal power flow (OPF). In the proposed model, the decision-dependency is included for the stochastic approach. The formulation of the decision-dependent probability distribution is based on the cost economic scale theory in electricity systems. The bilevel structure is recasted to a single level problem by taking advantage of the KKT optimal conditions. To further resolve the computation challenges and accelerate the calculation process, serval solution approaches are developed including linear transformation of the revenue term, linearization heuristics for decision-dependent probability and implantation for Dantzig-Wolfe decompostion. Extensive case studies are conducted based on IEEE reliability test systems. The study on a 3 bus electricity system shows that the multistage framework is able to advocate the growing demand of each stage and to minimize the construction cost. The comparison between the solutions for uncongested and congested network shows our model is able to take both sizing and siting into consideration. The study of computation times demonstrates the better performance by acquiring proposed solution algorithms. Finally, the computation result of VDDSS shows that it is important to take into account the decision-dependent approach for long-term investment planning problems.

In Chapter 4, we tackle the long-term power generation investment expansion planing problem by presenting a multistage adaptive robust optimization model. The multistage adaptive robust optimization model aims at finding the maximum profit by identifying the investment decisions that immunizes against all realizations of the uncertain data. We formulate the decision variable of generation capacity to be adjustable according to uncertain electricity demand. The simplified affine policy is adopted to restrict the decision rule of adjustable variables and therefore resolves the computation intractable issue. The computational experiments are conducted on both small test case and large-scale power systems to study the performance of the proposed model with comparisons to existing approaches. The results demonstrate the effectiveness the adaptive robust model in reducing the investment and maintenance cost, and at the same time improving the system reliability, compared to the existing stochastic programming approach and robust optimization approach.

The contributions of this dissertation can be summarized as follows,

1. This dissertation presents three multistage optimization models for the long-term electricity investment expansion planning problem with uncertainty. Each model studies the problem from different perspective. They all seek for the maximum profit by providing effective optimization information to the power investors.

2. This dissertation incorporates the decision-dependent probability distributions for each s-tochastic optimization models to faithfully model the real-world decision making process.

3. This dissertation develops solution approaches that incorporate transformation, decomposition, approximation and linearization techniques to resolve and accelerate the computation process for each optimization model.

4. This dissertation conducts extensive computational studies on the real-world large scale pow-er systems. From the numerical results, we discuss the merits the proposed models and the performances of solution algorithms.

5. Serval submitted/working journal papers are directly related to the work of this dissertation:

    (a) Y. Zhan, Q. Zheng, J. Wang, P. Pinson. A Decision Dependent Stochastic Programming Model for Power Generation Expansion Planning with Large Amounts of Wind Power, *IEEE Transaction of Power System*, 2016. DOI: 10.1109/TPWRS.2016.2626958

    (b) Y. Zhan, Q. Zheng. Bi-level Decision Dependent Stochastic Programming Model for Power Generation Investment Expansion Planning, *working paper*

    (c) Y. Zhan, Q. Zheng. Multistage Adaptive Robust Optimization for Power Generation Expansion Planning, *working paper*

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